Chapter 11:   WinTDR Algorithms

This chapter discusses the algorithms WinTDR uses to analyze waveforms including: Bulk Dielectric Constant; Soil Water Content; Electrical Conductivity; Calibrations for probe length and impedance; and Offset values. We will first discuss the steps used for analysis directly, then explain how we formulate reflection coefficient waveforms and equations we use for calibrations.

At the end of this chapter we explain the differences between sloped and flat analysis and give some examples of how we find the first and second derivatives.

WinTDR uses the following steps for analysis.
1. User positions the waveform in the window.
2. Pre-process: Program applies a moving average to the wave and finds the first and second derivatives of the wave.
3. Algorithm finds location of the **First Peak**.
4. Algorithm finds location of the **Second Reflection**.
5. Algorithm **uses the distance between** these two points to calculate the bulk dielectric constant (ε_b).
6. Algorithm uses Topp's equation to calculate the **soil water content**.
7. Algorithm uses an extended wave to calculate soil **electrical conductivity**.
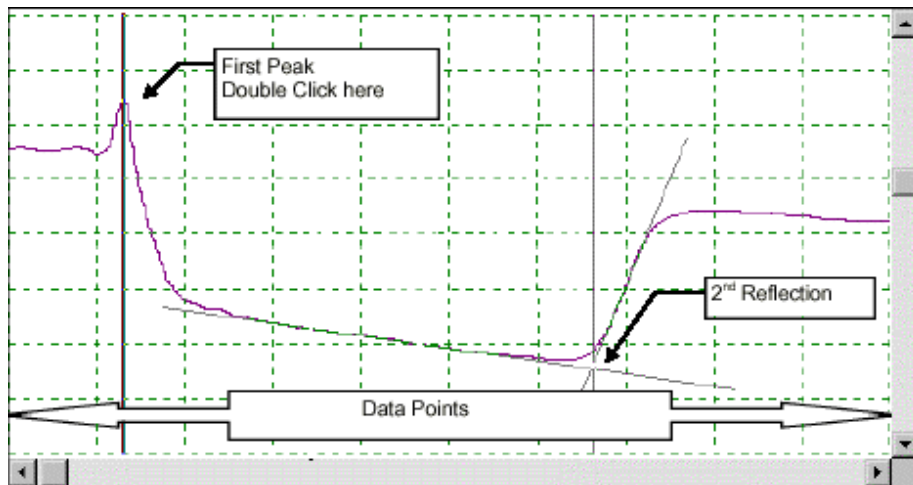

Figure 1: The **first Peak** and **Second Reflection** on data, which is *uniformly* captured by the data window.

Position the Waveform in the Window
- Position the waveform in the window. See **Figure 1**. For more information about positioning the waveform see Chapter 4.
- Set the **Peak Tolerance**. The *user defined peak tolerance* is a band around the user selected first peak location. It reduces the chances for failure. The first peak is usually constant, but changes in temperature may change the *cable bulk*

dielectric, causing slight variations. Defining a *first peak* and a *tolerance neighborhood* eliminates failures that may occur otherwise. (See **Figure 2**).

- The **Second Reflection** should be beyond the mid-point of the window for accuracy.

Pre-Process

- The wave is first smoothed using a moving average defined by the user. The moving average is normally set to zero. However, in some cases, there may be excess noise due to poor cable shielding and/or high amounts of interference. In these cases the user may set the moving average to a non-zero value which removes some of the noise for the analysis.
- Take its first derivative according to Eq.(1).

$$\frac{df}{dx} \cong \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2) \qquad \textbf{(1)}$$

- Take its second derivative by Eq.(2).

$$\frac{d^2 f}{dx^2} \cong \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2} \qquad \textbf{(2)}$$

Find the Location of the First Peak

- If the user has not defined a first peak, the *program* looks for candidate inflection points in the wave by scanning the first derivative and finding all candidate maxima. See **Figure E1-2.**
- If the user had defined a first peak, the *program* looks for candidate peaks within the defined peak tolerance. If none are found it uses the user defined peak as the calculated peak.
- The two largest peaks (in amplitude) will be the inflections on either side of the first peak. See **Figure E1-2**.

User defined *first peak* surrounded by user defined peak tolerance.

Tangents from the Inflections
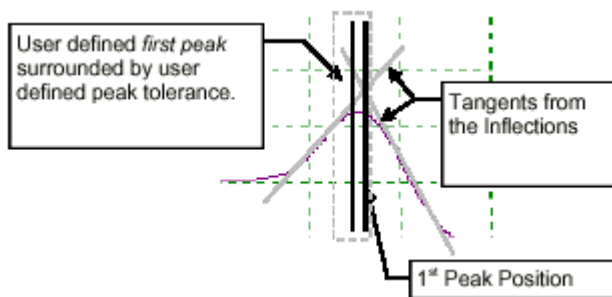
1st Peak Position

- Now it finds the true *first* peak, also called the *first reflection*.
- Two tangent lines from the inflections on either side of the candidate are intersected to create a *position* of the **first peak**.

Figure 2: Shows a waveform with a well-defined first peak, which will use the calculated first peak because it falls within the neighborhood.

Find the Location of the Second Reflection

The **second reflection** refers to the intersection of the tangent from the max *inflection* on the wave trace after the first peak, with either a tangent from the *local minimum* or a tangent of averaged slope from an anchored point between the *local minimum* and the first peak. See **Figure 3 and 4**.

- The **local minimum** in this case refers to the minimum encountered *after* the first peak.
- The **anchored point** refers to a point from 0 to 50% between the local minimum and the first peak. The exact placement defaults to 33% but may be changed by the user.
- The **max inflection** refers to the maximum inflection point after the first peak.

The two tangent lines are intersected. Where these two lines intersect can be called *location 2*, or *the Second Reflection*. *Location 1* is the first peak. Their difference (location 2 – location 1) is used to directly calculate the **bulk dielectric constant** (ε_b). See **Equation 3** below.

Some difficulty can be encountered in finding *location 2* when the soil is saline, very wet and with a high clay content. Signal attenuation may be so strong that the second reflection is lost and the ability to determine water content is also lost. This happens at about EC levels > 3 dS/m in soil. Frequency domain analysis is sometimes used to determine water content when the second reflection of the wave cannot be located.

Figure 3: A three pronged probe immersed in Tap Water (Ec=03 dS/m), displays virtually no difference in the second reflection calculation between the tow methods. This is because the rising limb dominates the intersection.

To better locate this second reflection, within soils with moderate (low attenuation) salinity, a line not from the local minimum, but rather from the slope before the local minimum is used. This method is called the **sloped line** method. See **Figure 3 & 4** for a graphical explanation of the differences between the two methods.

Otherwise, if the waveform is relatively uniform, the second reflection can be found using the **flat line** method. The user should use judgment in choosing which method to use. WinTDR default is the flat line method.

Summary of Finding the Second Reflection

Two slightly different methods of finding the *Second Reflection* can be used:
- Flat line method (designed for well defined waveforms)
- Sloped line method (designed for waveforms from soil of high salinity or clay content).
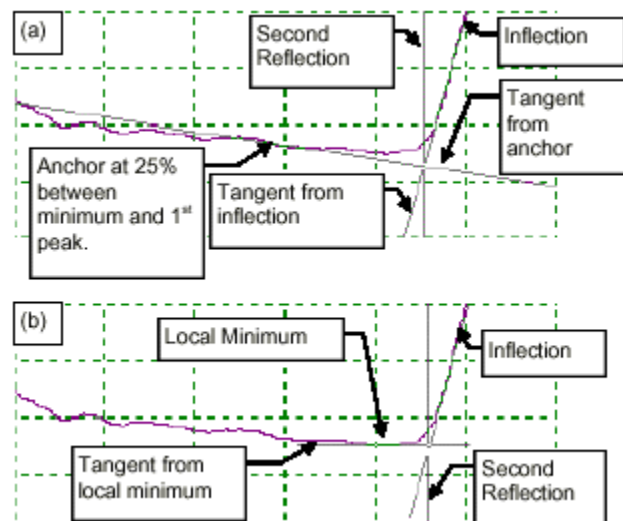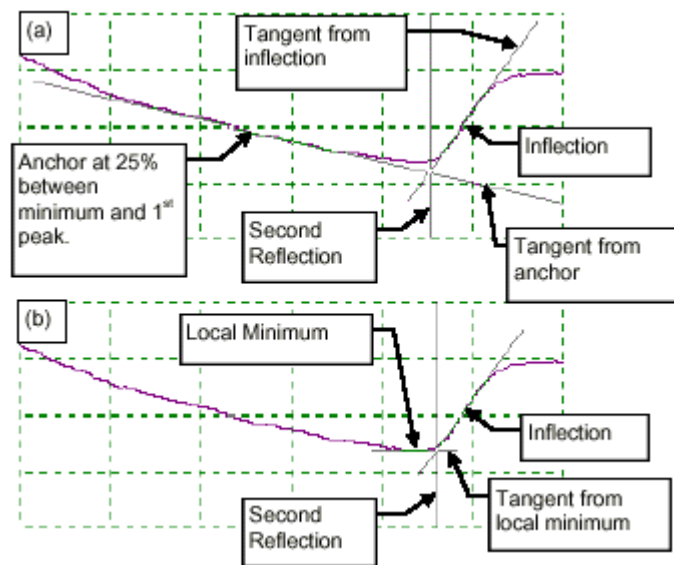
Figure 4: A three-prong probe immersed in **saline solution** (Ec=1.8 dS/m). **(a)** Shows a **tangent from an anchor** point 25% between the first peak and the *max inflection*. **(b)** Shows a **tangent from the local minimum**. Both tangents intersect the tangent from the inflection point. This intersection is the **second reflection**. Notice the difference, (a) intersects slightly to the left of (b).

Computation of Water Content Using Equations for Bulk Dielectric Constant and Topp's (1980) Empirical Expression

Compute the bulk dielectric constant, using:

$$\varepsilon_b = \left(\frac{c}{v}\right)^2 = \left(\frac{cT}{2L}\right)^2 \tag{3}$$

Where:
  c = the speed of light
  v = velocity of propagation
  L = length of TDR probe
  T = travel time for the pulse to traverse the length of the embedded wave guide

v Is usually set to 0.99, but can be changed in the program.
T Is found from the difference between time 1 and time 2 (converted to time from the first peak and second reflections).
L Is the measured length of the TDR Probe of calculated from measurements using the TDR Probe.

Calculate Water Content

The computed bulk dielectric constant (ε_b) is then used in Topp's (1980) equation to calculate the volumetric water content according to:

$$\theta_v = -5.3x10^{-2} + 2.92x10^{-2}\varepsilon_b - 5.5x10^{-4}\varepsilon_b^{\,2} + 4.3x10^{-6}\varepsilon_b^{\,3} \qquad \textbf{(4)}$$

The user may change the coefficients of Equation 4 to adjust the calculation and calibrate for individual soil samples.

Calculate Electrical Conductivity

Calculating Ec uses the Giese and Tiemann (1975) method given by Eq. **(5)**

$$Ec = \left(\frac{\varepsilon_0 c}{L}\right)\left(\frac{Z_0}{Z_u}\right)\left(\frac{2V_0}{V_{inf}} - 1\right) \quad \text{(S/m)} \qquad \textbf{(5)}$$

Where:
- ε_0 = Permittivity of free space (8.854 x 10^{-12} F/m).
- c = The speed of light in a vacuum (2.99792458 x 10^8 m/s).
- L = Probe Length (m).
- Z_0 = Characteristic Probe Impedance. (Ω) (Must be obtained for each probe by calibration. See "Calculate Characteristic Impedance" below)
- Z_u = TDR cable tester load impedance (typically 50 Ω)
- V_0 = Incident pulse voltage (Obtained from the collected waveform. See **Figure 5** below.)
- V_{inf} = The return pulse voltage after multiple reflections have died out. (Obtained from the collected waveform. See **Figure 5** below.)

And Since ε_0 c = 1/(120π), Eq. **(5)** is simplified as:

$$Ec = \frac{Z_0}{120\pi L Z_u}\left(\frac{2V_0}{V_{inf}} - 1\right) \qquad \text{(S/m)} \qquad \textbf{(6)}$$

To find V_0 and V_{inf} we require an extra waveform that demonstrates the long-term behavior or the medium. We call this waveform a long wave. This waveform has a distance per division of 5 meters and begins 5 meters before the user defined cursor position (or 0.1 meters which ever is greater). Returned is a waveform similar to Figure 5.
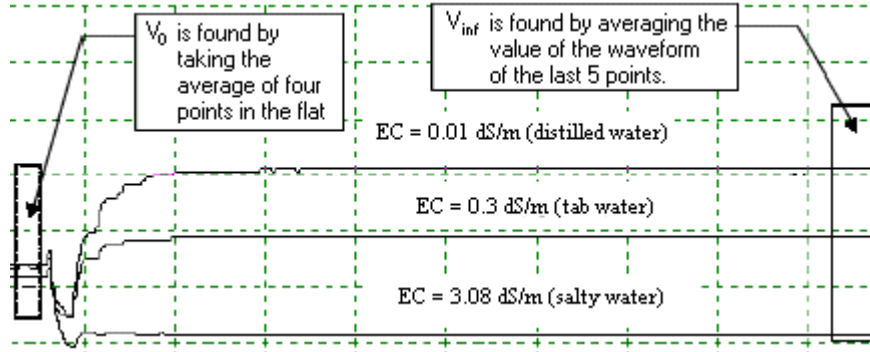
Figure 5: How V_0 and V_{inf} are acquired.

Once V_0 and V_{inf} are acquired, we now can plug each variable into Equation 2 to find the Electrical conductivity of the medium.

Now we will discuss how we formulate reflection coefficient values and how we calibrate length and impedance.

Formulating Reflection Coefficient values (Rho)

To calculate Rho values we use Eq. **7**.

$$RhoValue = \frac{P - V_0}{V_0 - Z_0} \qquad (7)$$

Where:
P   = Waveform Pixel Value.
Z_0 = The TDR Connector reference voltage. This is found by obtaining a waveform of –0.6 meters and averaging the first 10 waveform pixel values. It will change depending on the medium the probe is in.
V_0 = This value is the same as described in Ec calculation (Figure 5), except note that it is a pixel value, not a rho value.

When we "Re-Offset" Probe, we retrieve a new waveform from –0.6 meters with distance per division of .5m and average Z_0 and V_0 again. It is important to re-offset if the probe medium is changed.

Calibrating Probe Length

Calibrating the probe length is simply manipulating Eq. 3 to arrive at Eq. **8**.

$$L = \frac{cT}{2\sqrt{\varepsilon_b}} \qquad (8)$$

If the user provides an expected bulk dielectric for the solution, we can calculate the effective length of the probe using Eq. 8. (cT actually is the second reflection - peak position).

Calibrating Characteristic Impedance

To calibrate characteristic impedance we use Eq. **9**.

$$Z_0 = Z_u \sqrt{\varepsilon_b} \left(\frac{1 + V_1}{1 + 2V_0 - V_1} \right) \tag{9}$$

Where:

ε_b = Bulk Dielectric
Z_u = TDR cable tester load impedance (typically 50 Ω)
V_0 = Incident pulse voltage (Obtained from the collected waveform. See **Figure 5**.)
V_1 = Minimum before second reflection.

Differences Between Slope and Flat Method

There are virtually no differences between the flat and slope method. For illustration purposes consider the following test:

- Three pronged TDR probes are embedded in two soil bins – one wetted with tap water (**Normal**) and the other with salt solution (**Salty**). The soil type in both bins in Millville silt loam. The electrical conductivity (Ec) of tap water was 0.3 dS/m, whereas the Ec of the CaCl+2 solution was 5.3 dS/m. The two soil bins were brought to saturation and allowed to dry by means of evaporation.
- One document was opened in WinTDR and readings taken every fifteen minutes for both of these soil bins. This document was then saved. A duplicate document was created by using the same setup file, (**Save as** command). One was set to take readings using the *flat line* method and the other the *sloped line* method. The results show that no significant difference occurs between the two methods, especially over a long period of time. See **Figure 5**. (NOTE: In WinTDR 6.0 Beta, you would have to change the analysis method each time analysis was done. It does not support multiple analyzer's.)
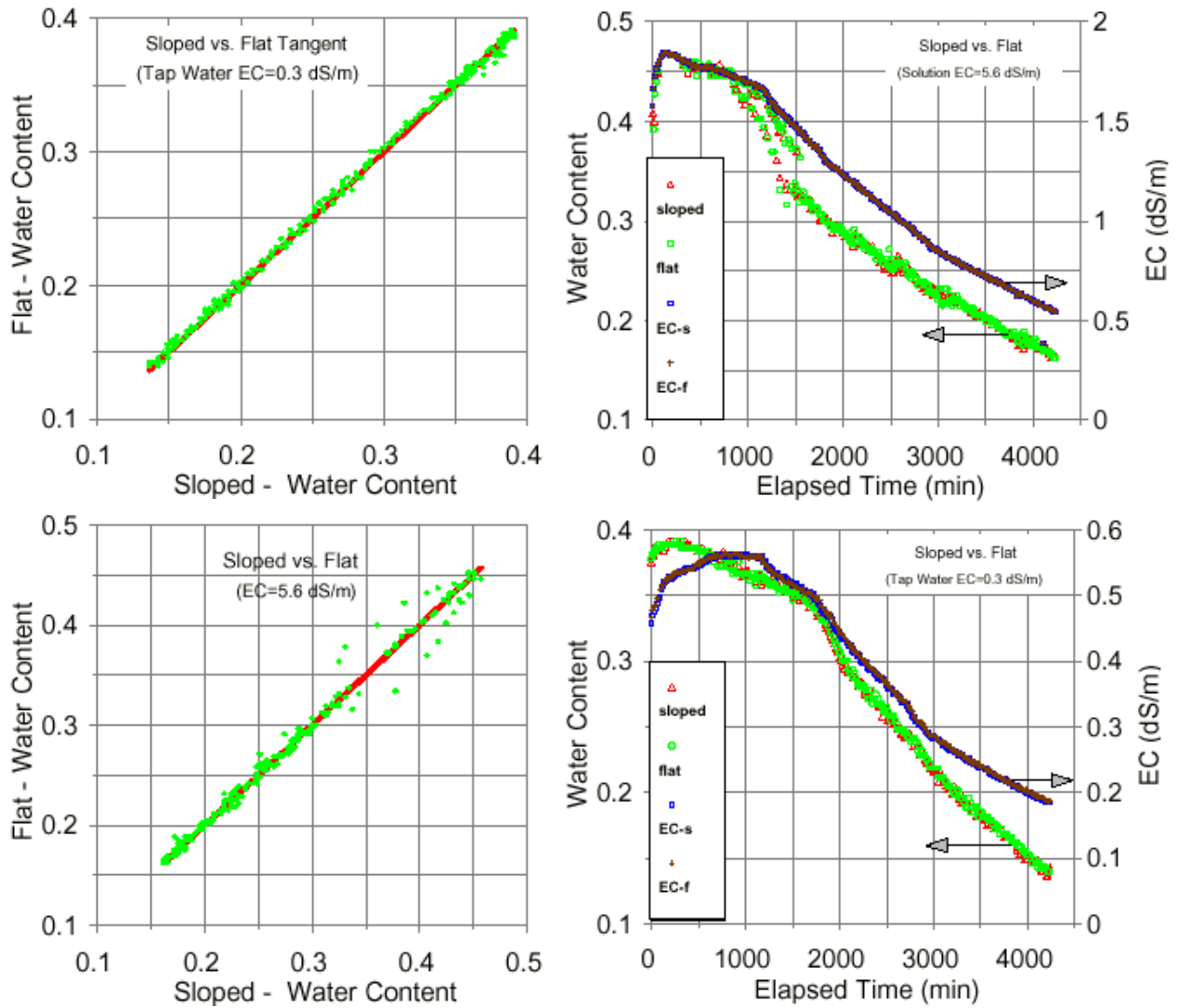
Figure 6: From Top-Left Clockwise: Slope vs. Flat Tangent method (Ec = 0.3 dS/m); Time vs. Water Content and Ec, (Ec = 0.3 dS/m); Time vs. Water Content and Ec (Ec = 5.6 dS/m); Sloped vs. Flat Tangent method (Ec = 5.6 dS/m).

As **Figure 6** illustrates, the *Flat Line* and the *Sloped Line* methods produce similar interpretations of soil water contents (even under salty conditions)…

Example: Finding the first derivative.

Some may consider trying the following experiment on their own.
1. **Acquire a well-defined waveform** using the WinTDR software.
2. Save the waveform to file.
3. Import the waveform to a good spreadsheet application.
4. Graph the points of the acquired waveform, you should see something like **Figure E1-1.**
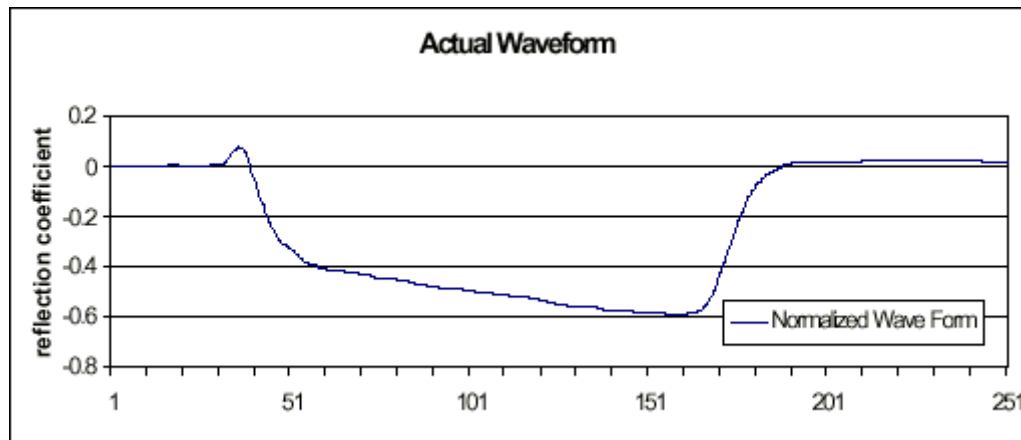

Figure E1-1: An actual waveform created by importing the points from a WINTDR wave output file to a Microsoft Excel Spreadsheet. Notice how well it is positioned with the *first peak* near the left side of the screen, and the *second reflection* near the right side of the screen.

6. Take the first derivative. Using the previous wave trace in the spreadsheet, create a formula that acts on each of the points that reads "=(h1-f1)/2" where the two cells referenced are the cells directly on each side of the cell in question.
   Note: WinTDR allows the user to choose how many points on each side. This allows increased accuracy if wanted. By default WinTDR uses the points three away on each side.
7. Recover the lost points on the ends by copying the values of their closest neighbor. Near the ends, this *error* is very insignificant. WinTDR doesn't use these end values for any calculation, but it will make your graph more presentable.
8. Plot the first derivative. See **Figure E1-2**.
9. Take the second derivative by applying the same algorithm to the first derivative. Graph this to get **Figure E1-3**.

Note: This shows the slope crossing the x-axis at approximately point 35. In your spreadsheet, the exact figure can be seen where the slope crosses the x-axis. **Figure E1-3** also shows the inflection that brackets the first peak. WinTDR uses the intersection of the tangents from the inflections on either side of the first peak, as the calculated first peak.
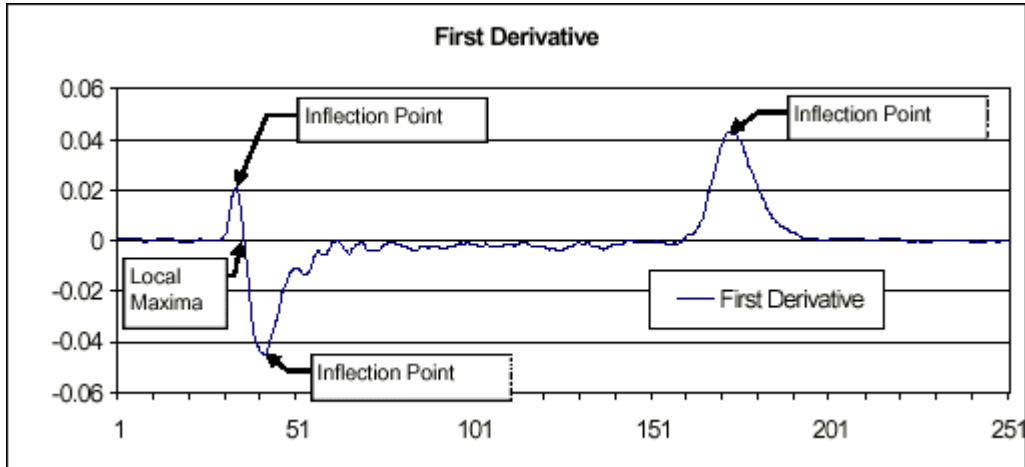
Figure E1-2: Shows the **first derivative** of the wave trace. Notice the slope hits zero at approximately point 36.
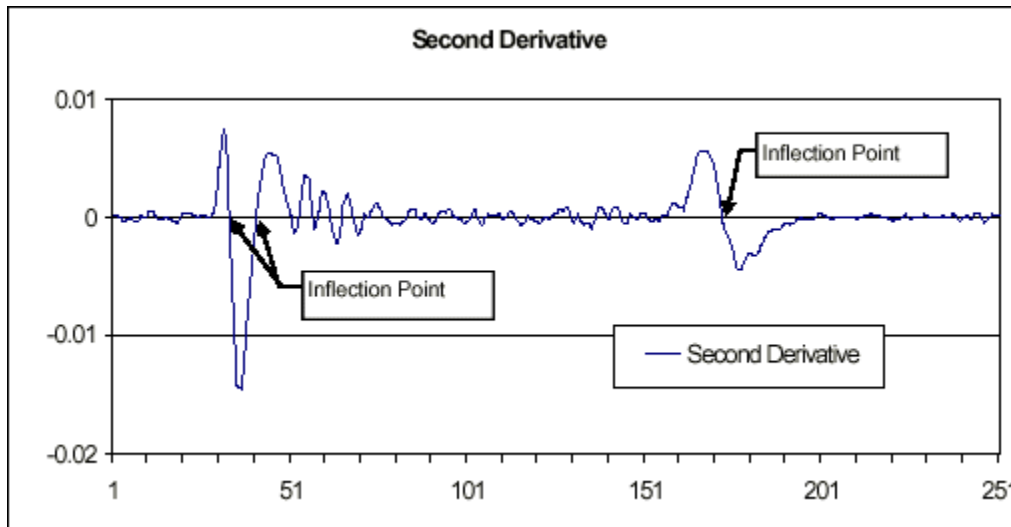

Figure E1-3: Shows the **second derivative**. The inflection points are clearly visible as the points of crossing the x-axis.

Electrical Conductivity Calculations Using the Castiglione and Shouse (2003) Method:

The Castiglione and Shouse (2003) method for calculating electrical conductivity takes into account cable and connector losses that occur along the transmission line by scaling reflection coefficients with reference to reflection coefficients measured in air and with the probe short-circuited. The waveforms in air and with the shorted-circuited probe represent an envelope that the subsequently measured media fall between (see Figure 6). For highest electrical conductivity measurement accuracy, it is recommended that the Castiglione and Shouse (2003) method be used.
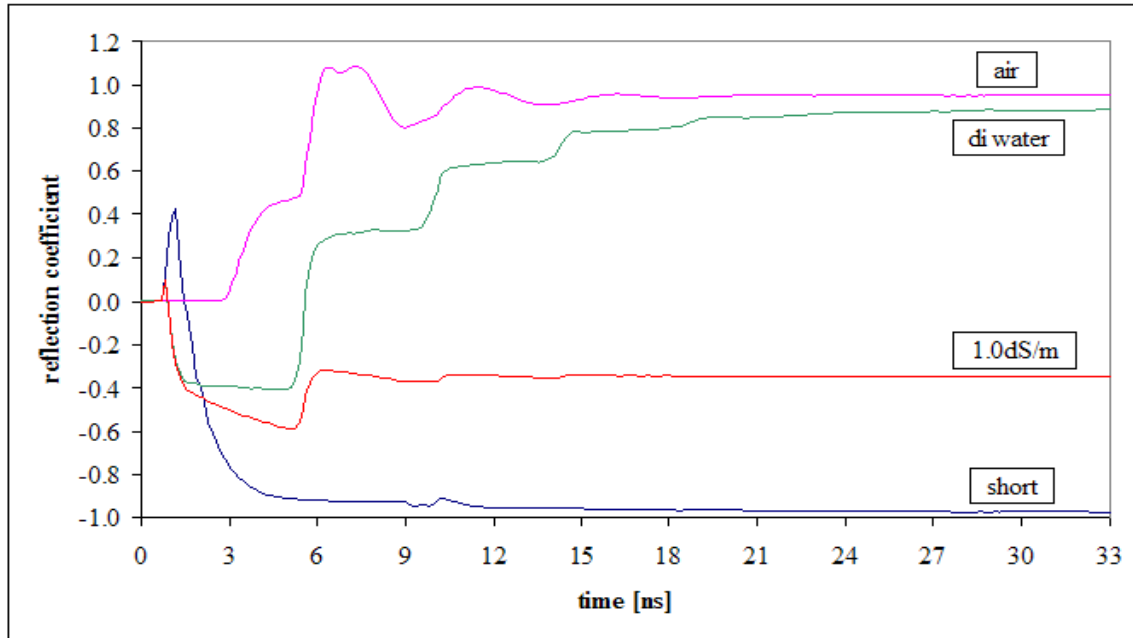
Figure 6: Waveforms in air and with the probe short-circuited form an envelope that all subsequent measured waveforms fall between. Here waveforms taken in de-ionized water and in a 1.0dS/m solution are shown for comparison.

Electrical conductivity calculations (Ec) use the Giese and Tiemann (1975) method written in terms of reflection coefficient (written in terms of voltage previously) given by equation 10:

$$Ec = \left(\frac{\varepsilon_0 c}{L} \right) \left(\frac{Z_0}{Z_u} \right) \left(\frac{1 - \rho_{scaled}}{1 + \rho_{scaled}} \right) \quad [\text{S/m}] \tag{10}$$

Where:
      ε_0=permittivity of free space (8.854×10^{-12} F/m).
      c=speed of light in vacuum (3.0×10^8 m/s).
      L=length of the probe.
      Z_0=characteristic probe impedance (Ω) (determined by calibration, discussed below).
      Z_u=characteristic cable impedance (equal to 50Ω).
      ρ_{scaled}=reflection coefficient after multiple reflections have died out and scaled in reference to reflection coefficients in air and with the probe short circuited (scaling reflection coefficients is discussed below).

Probe Impedance Calibration Using the Castiglione and Shouse (2003) Method:
Characteristic probe impedance is determined by calculating electrical conductance (G) values from scaled infinite reflection coefficients which are taken from waveform(s) measured in sample(s) of known Ec (determined with an Ec meter). Conductance is calculated according to equation 11.

$$G = \left(\frac{1}{Z_u}\right)\left(\frac{1-\rho_{scaled}}{1+\rho_{scaled}}\right) \quad [S] \tag{11}$$

The calculated conductance values are then plotted as a function of the known Ec values and a regression line is fitted to the point(s) and forced through the origin. The inverse of the slope of the line is the cell constant (K_p) according to equation 12:

$$G = Ec / K_p \quad [S] \tag{12}$$

The $\varepsilon_0 c Z_0 / L$ term in equation 12 is equivalent to the probe constant and is given by equation 13:

$$K_p = \frac{\varepsilon_0 c Z_0}{L} \quad [m^{-1}] \tag{13}$$

The probe impedance is now calculated by rearranging equation 13 and using the calibrated cell constant to solve for Z_0, which is then used as a constant (for the particular probe) in equation 10 to calculate Ec. NOTE: Probe impedance values calculated using this method are specific to the probe/cable/connector/multiplexer setup used during calibration. If this setup is changed a new probe cell constant and impedance value should be calculated for the new setup.

Scaling Reflection Coefficients for Use in the Castiglione and Shouse (2003) Method:
Reflection coefficients in WINTDR are determined from an average of 10 discrete waveform points (calculated using equation 7 described above). Infinite reflection coefficients are located at approximately 600 m. The reflection coefficient is scaled with reference to measured air and measured short-circuited probe reflection coefficients in order to account for cable, connector, and multiplexer signal voltage losses. Reflection coefficients are scaled according to equation 14:

$$\rho_{scaled} = 2\left(\frac{\rho_{inf} - \rho_{air}}{\rho_{air} - \rho_{sc}}\right) + 1 \tag{14}$$

Where:

ρ_{inf}=infinite reflection coefficient from waveform measured in solution of known conductivity.

ρ_{air}=infinite reflection coefficient from waveform measured in air.

ρ_{sc}=infinite reflection coefficient from waveform measured with the probe short circuited (instructions on short circuiting probes are given below).

Short Circuiting Probes

Probes can be short circuited by clamping a conductor, such as copper, between the prongs of the probe, or inserting two of the probe prongs into a copper pipe. Figure 1 shows examples of a probe being short circuited.
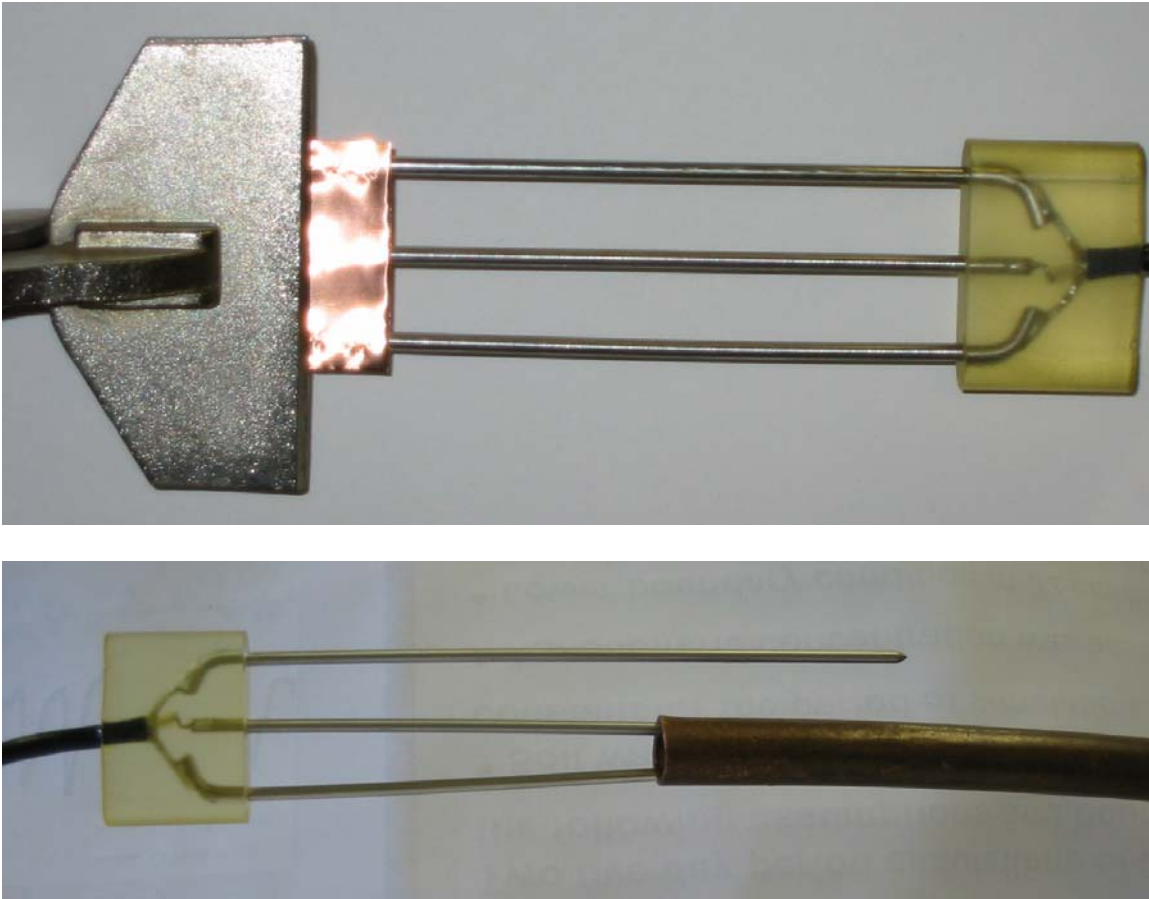

Figure 7: (a) Probe short-circuited with a thin copper sheet and flat (duckbill) vicegrip pliers and (b) probe shorted circuited with a copper pipe.