

Hydraulics Overview

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July 31, 2024

- General observation & some terms
- Conservation of mass & force balance
- Flow resistance – *what sets the depth?*
- Using all the tools at once
- Energy in open channel flow
 - Rapidly varied & gradually varied flow
- Water surface, or backwater modeling



The open channel toolbox

(for flow and sediment transport)

- Conservation Relations

 - Water Mass (aka continuity)

 - Momentum (aka $F = ma$, *Newton's 2nd Law*)

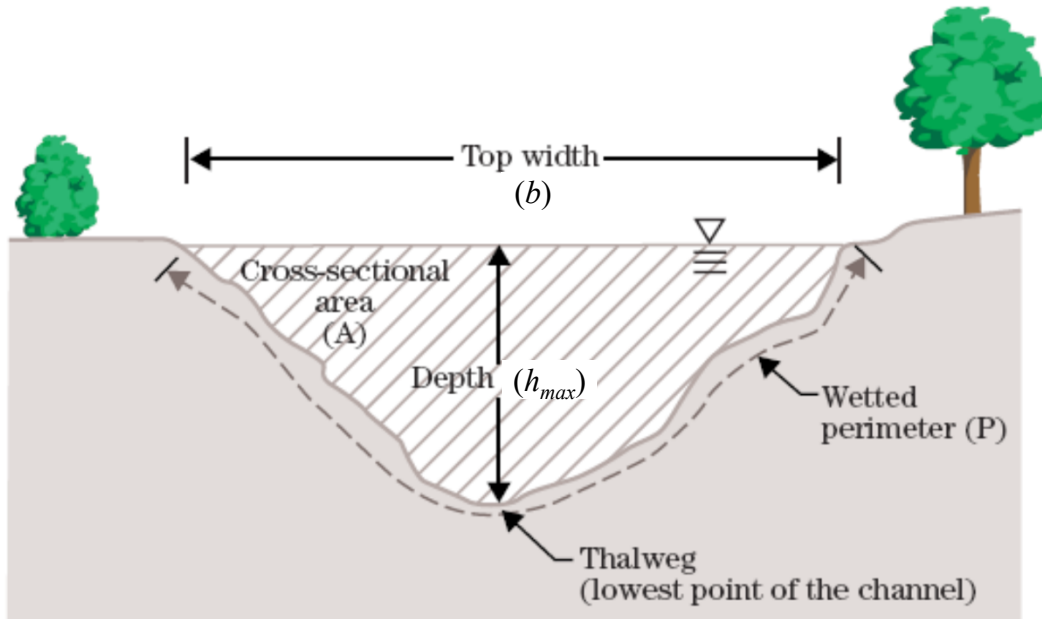
 - Energy

- Constitutive Relations

 - Flow resistance (we'll use Manning eq.)

 - Sediment transport (Wednesday)

- Morphodynamics – add conservation of sediment mass
(Wednesday/Thursday)



STEADY flow: not changing in time $\frac{\partial}{\partial t} = 0$

UNIFORM flow: not changing in space $\frac{\partial}{\partial x} = 0$

NORMAL flow: steady and uniform (!)

- Distorted view
(channels are much wider than they are deep;
 $b/h = 20$ is a narrow channel)

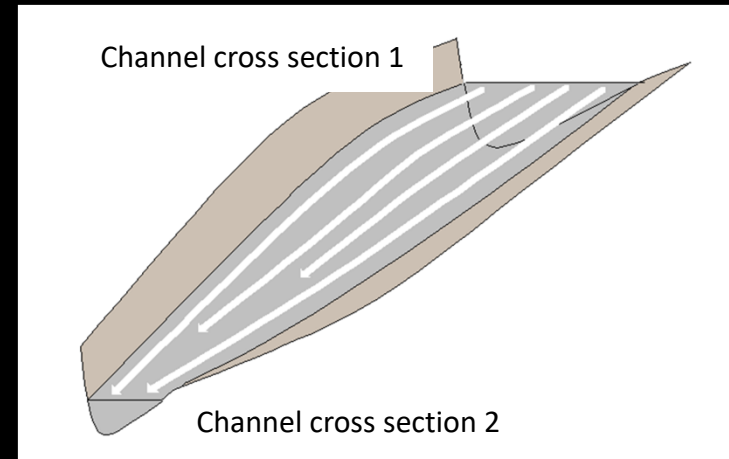
1:20

- Width b , area A , wetted perimeter P
- Hydraulic radius $R = A/P$
- Mean depth $h = A/b$
- Often, $b \approx P$, so $R \approx h$
e.g. for a rectangular channel

$$R = \frac{bh}{b+2h} \approx \frac{bh}{b} = h$$

Continuity

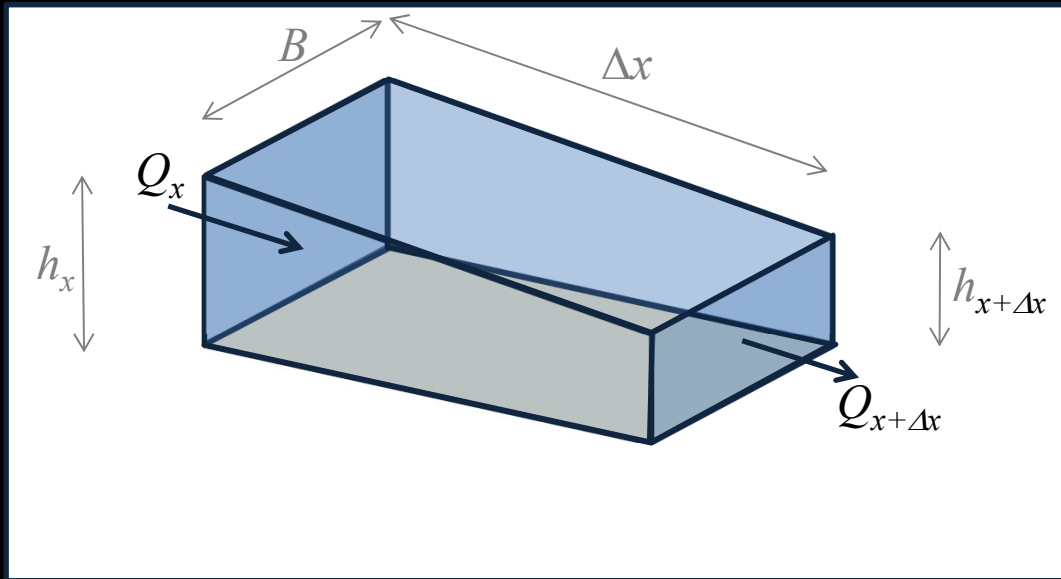
- Conservation of water mass
- Just accounting, like your checkbook
- Rate of change of water mass in reach = net rate of input and output
- Inputs and outputs:
- Constant discharge: input = output, no change in water mass in reach



$$Q_1 = U_1 A_1 \quad \text{and} \quad Q_2 = U_2 A_2$$

$$\text{For } Q_1 = Q_2, \quad \frac{U_2}{U_1} = \frac{A_1}{A_2}$$

Q is water discharge (e.g. ft³/s or m³/s); U is mean velocity (e.g. ft/s or m/s)



Continuity for unsteady and nonuniform 1d flow –

Now depth h varies in both time and space

$$\Delta S = I - O \longrightarrow$$

For a control volume,

The rate of change of mass = the net rate of mass flowing through the sides

$$\frac{\partial \rho \nabla}{\partial t} = \rho U h B|_x - \rho U h B|_{x+\Delta x} = -\frac{\partial(\rho U h B)}{\partial x} \Delta x$$

For constant ρ, B .
$$\frac{\partial h \Delta x}{\partial t} = -\frac{\partial(Uh)}{\partial x} \Delta x$$

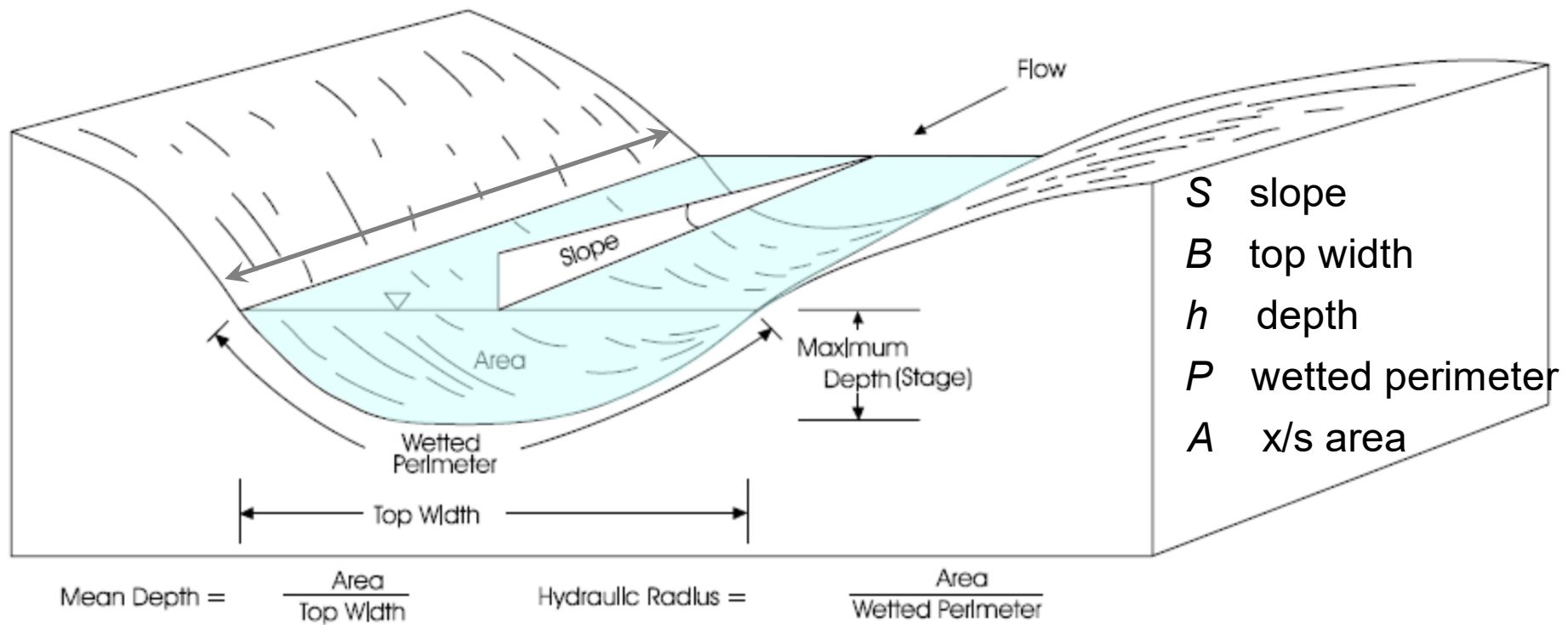
or
$$\frac{\partial h}{\partial t} = -\frac{\partial(Uh)}{\partial x}$$

$$\frac{\partial h}{\partial t} + \frac{\partial(Uh)}{\partial x} = 0$$

If $\partial/\partial t = 0$, we recover $UhB = Q = \text{constant}$

Momentum: Newton's 2nd Law, $\Sigma F = ma$

Normal flow
no accelerations, so $\Sigma F = 0$



Volume of water: AL
 Weight of water: ρgAL
 Downslope component of
 weight of water: $\rho gALS$

Boundary stress: τ_o
 (stress is force/area)
 Boundary force: $\tau_o PL$



τ_o is the boundary shear stress -
 the flow force per unit area -
 it drives the sediment transport

$$\tau_o PL = \rho gALS$$

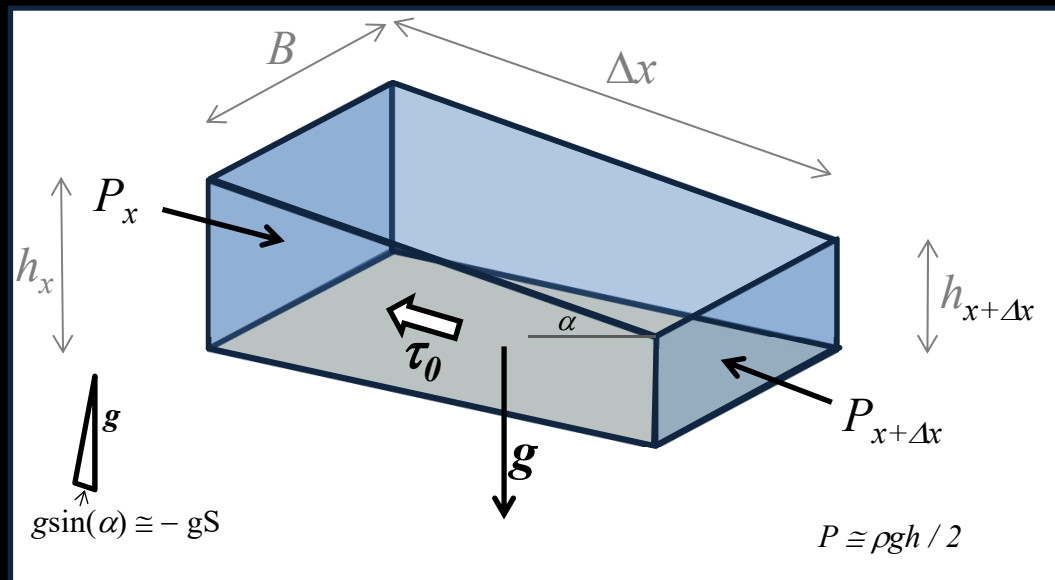
$$\tau_o = \rho gRS$$

The 'depth-slope' product
 $\tau_o \approx hS$ (h in cm, S in %, τ_o in Pa)

1b. Boundary shear stress — Unsteady, Nonuniform Flow

We still use Newton's second law, $\Sigma F = ma$

But the flow is accelerating in (1) space and (2) time.
These accelerations are balanced by the sum of forces, to which we now must add a third force, due to the difference in flow depth down the reach



If the flow is predominantly one-dimensional (no rapid changes in width or depth that give rise to vertical or lateral accelerations), we get the

1d St. Venant Eqn:

$$m\mathbf{a} = \mathbf{F}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = gS - g \frac{\partial h}{\partial x} - \frac{\tau_o}{\rho R}$$

An approximate derivation is in the primer

“local” acceleration “convective” acceleration body force pressure force boundary force

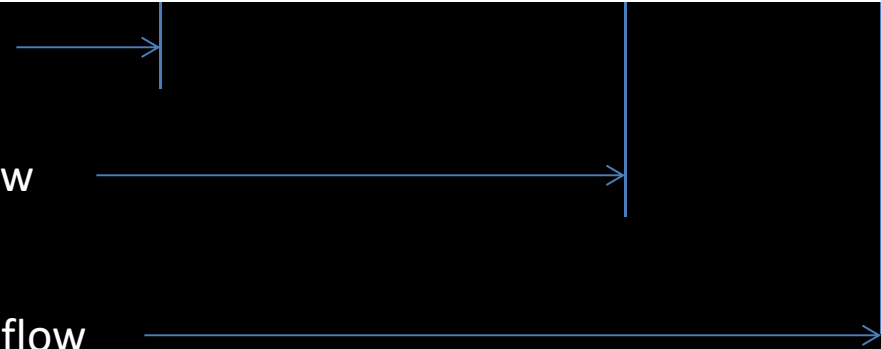
Rearrange to solve for τ_o

$$\tau_o = \rho g R \left(S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right)$$

Steady, uniform flow

Steady, nonuniform flow

Unsteady, nonuniform flow



$$\tau_0 = \rho g R \left(S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right)$$

1. For steady, uniform flow, we recover the depth-slope product
2. But flow is never perfectly steady and nonuniform!
3. So, how do we know whether the other terms are important or not?
4. Are we making an *assumption* or an *approximation*?

NOTE:

$$\tau_0 = \rho g R \left(S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right) = \rho g R S_f$$

Backwater programs

(HEC-RAS) compute:

$$S_f = \frac{d}{dx} \left(z_b + h + \frac{U^2}{2g} \right)$$

→ [flood.xls](#)

Flow from a tributary at point a increases rapidly from Q_1 to Q_2 over Δt . At time Δt , any increase in flow has not yet reached a distance Δx downstream, creating a nonuniform flow 'wedge'. The flow is unsteady (it is increasing with time) and it is nonuniform (it is increasing upstream).

Question: do the unsteady and nonuniform terms contribute significantly to τ_o ?

Given a rectangular channel:		The discharge at pt a increases	
Channel width b	15 (m)	from Q1 to Q2 over Δt	
Channel slope S_o	0.001	Q1	10 (m ³ /s)
hydraulic roughness n	0.035	Q2	15 (m ³ /s)
		Δt	3600 (s)



Note that a number of cells are named and that those names are used in formulas

Extra Challenge

Enter Data only in green cells
 Flow is unsteady and nonuniform, so we ask which terms in the St Venant Eqn are significant
 How does the shear stress change, approximately, over this reach of nonuniform flow?
 >>> we need to estimate the magnitude of the different terms in the St Venant Eqn.

$$\tau_o = \rho g R \left[S_o - \frac{dh}{dx} - \frac{U}{g} \frac{dU}{dx} - \frac{1}{g} \frac{dU}{dt} \right]$$

$$\tau_o = \rho g R \left[S_o - \frac{\Delta h}{\Delta x} - \frac{U}{g} \frac{\Delta U}{\Delta x} - \frac{1}{g} \frac{\Delta U}{\Delta t} \right]$$

To make approximation, express in difference form

The length of the flood wave Δx is found from $U_f = \Delta x / \Delta t$, where U_f is the speed of the flood wave and is approximately 1.5 times mean velocity in the reach

Δx 4470 (m)

We need to determine depth and velocity at the upstream and downstream end of the reach

UPSTREAM			DOWNSTREAM		
h_u	1.12	(m)	h_d	0.87	(m)
R_u	0.98	(m)	R_d	0.78	(m)
U_u	0.89	(m/s)	U_d	0.77	(m/s)

$$U = \frac{\sqrt{S_o}}{n} R^{2/3} \text{ and } Q = UA$$

so, for a rectangular channel ($A = bh, P = b + 2h$)

$$Q = \frac{\sqrt{S_o}}{n} \frac{(bh)^{2/3}}{(b + 2h)^{2/3}} bh \text{ or}$$

$$h = \frac{1}{b} \left(\frac{nQ}{\sqrt{S_o}} \right)^{3/5} (b + 2h)^{2/5}$$

which can be solved iteratively for h

We also need the mean velocity and hydraulic radius for the reach

R	0.88	(m)
U	0.83	(m/s)
Δh	-0.253	(m)
ΔU	-0.12	(m/s)

Now we can compute all of the components in the brackets of the St Venant eqn

Channel slope S_o	0.00100
$\Delta h / \Delta x$	-0.00006
$(U/g) * (\Delta U / \Delta x)$	0.00000
$(1/g) * (\Delta U / \Delta t)$	0.00000
Σ	0.00106

drop if any of these terms are more than an order of magnitude smaller

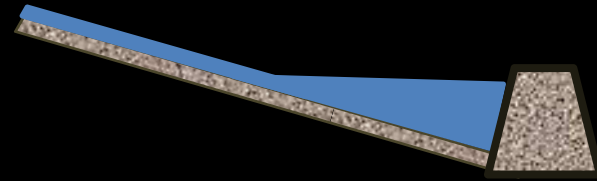
Mean τ_o in nonuniform reach, approximated from St Venant eqn.

τ_o 9.16 Pa

τ_o in steady uniform flow before flood

7.65 Pa

$$\tau_0 = \rho g R \left(S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right)$$



Use St. Venant to consider sediment transport for a steady river flow entering a reservoir. Assume a wide channel, so $R = h$. As flow enters the reservoir, it deepens and slows (by continuity!). Which effect wins out?

$$\tau_0 = \rho g h \left(S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$$

Does boundary stress τ_0 increase or decrease as we enter the reservoir?

Consider $\left(-\frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$ for the case of constant width b (for simplicity)

such that $q = Uh = \text{const}$ and $\frac{\partial q}{\partial x} = 0 = \frac{\partial(Uh)}{\partial x} = U \frac{\partial h}{\partial x} + h \frac{\partial U}{\partial x}$ such that

$\frac{\partial U}{\partial x} = -\frac{U}{h} \frac{\partial h}{\partial x}$. Replace $\frac{\partial U}{\partial x}$ in the first equation above, giving

$\left(-\frac{\partial h}{\partial x} + \frac{U}{g} \frac{U}{h} \frac{\partial h}{\partial x} \right) = \frac{\partial h}{\partial x} \left(\frac{U^2}{gh} - 1 \right)$. Because $\frac{U^2}{gh} < 1$ in a reservoir,

the entire quantity $\left(-\frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$ is negative. Because $\frac{\partial h}{\partial x}$ is itself positive

for flow entering a reservoir, we see that τ_0 is reduced from $\rho g h S$.

A smaller value of τ_0 means that transport capacity decreases and sediment deposits.

← The answer

Consider a typical problem:

Specify a channel, with known geometry

(fluid ρ and acceleration of gravity g are also known).

For a specified discharge Q , what is the flow depth h for steady uniform flow?

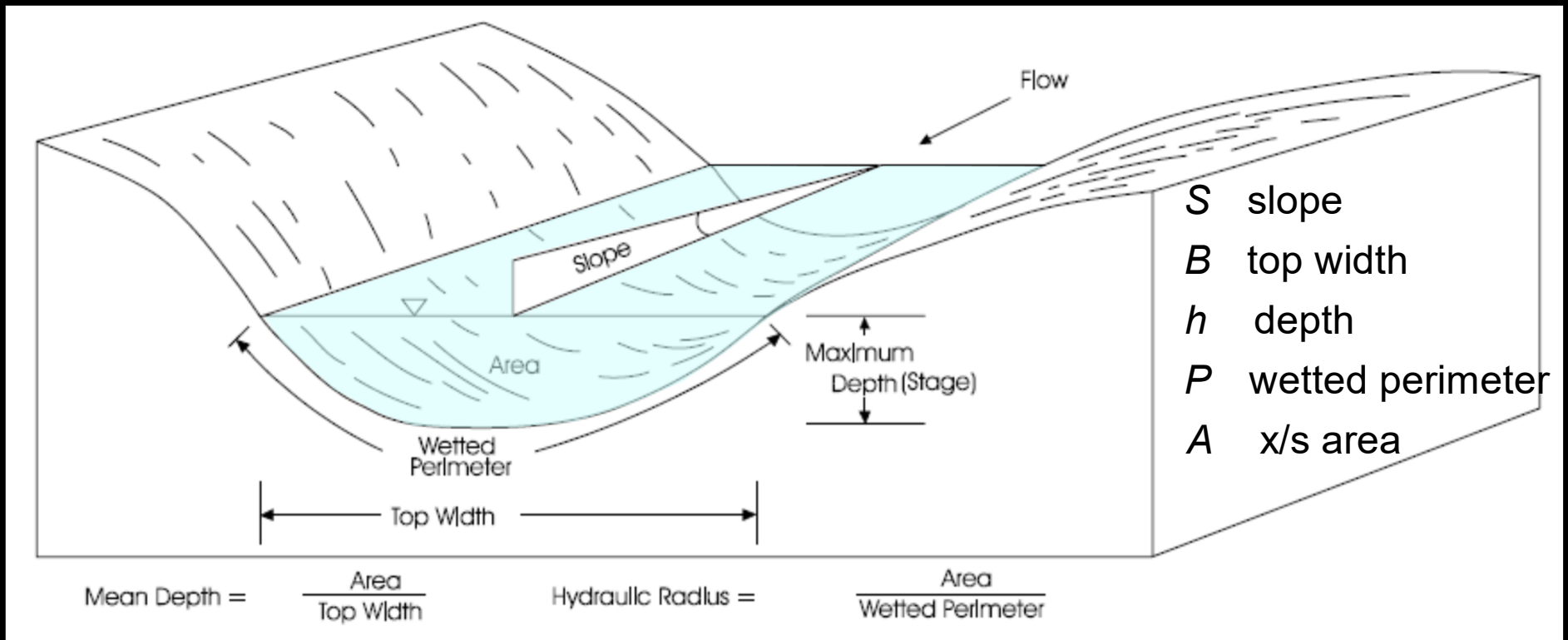
{when h is known, channel geometry gives is area A and hydraulic radius R }

Continuity gives us U (when $A(h)$ is known).

Momentum gives τ_o (when $R(h)$ is known).

Energy gives $S_f = S_o$ for steady uniform flow.

We need one more relation to find depth h ! \rightarrow flow resistance (a constitutive relation).



Flow Resistance

- A relation between velocity, flow depth, boundary stress, and boundary roughness
- We'll use Manning's eqn.
- Prefer an eqn. with constant roughness (n)
- Using continuity:

$$U = \frac{\sqrt{S}}{n} R^{2/3}$$

$$Q = UA = \frac{\sqrt{S}}{n} AR^{2/3}$$

S is the slope of the 'energy grade line', which is also the slope of the channel for uniform flow

- For a useful approximation, consider a wide ($R \approx h$) rectangular channel
- Solve for depth
- In field, relation between Q , A , & R more complex, though not more complicated
- In practice, a flow resistance relation is often used to find the flow depth for a specified flow in a given channel

$$Q = \frac{\sqrt{S}}{n} AR^{2/3}$$

$$Q = \frac{\sqrt{S}}{n} (bh)h^{2/3}$$

$$Q = \frac{\sqrt{S}}{n} bh^{5/3}$$

Manning's Eq., 3 ways:

$$n = \frac{b\sqrt{S}h^{5/3}}{Q}$$

$$Q = \frac{b\sqrt{S}h^{5/3}}{n}$$

$$h = \left(\frac{nQ}{b\sqrt{S}} \right)^{3/5}$$

The hard part: determining roughness ' n '

- How to measure:
survey cross section and reach, which gives S and A , R as a function of water surface elevation (aka stage or WSEL)
- Measure discharge at a known stage, solve for n

$$Q = \frac{\sqrt{S}}{n} AR^{2/3}$$

$$n = \frac{\sqrt{S}}{Q} R^{2/3} A$$

- If you don't measure, you are guessing
- Picture books, tables
- Formulas $n = \text{fn}(D, R, \text{veg.}, \dots)$

Roughness Characteristics of Natural Channels

By HARRY H. BARNES, Jr.

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1849

Color photographs and descriptive data for 50 stream channels for which roughness coefficients have been determined



UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1967

$n = 0.045, 0.073$

10-1550. Provo River near Hailstone, Utah

Gage location.—Lat $40^{\circ}36'$, long $111^{\circ}22'$, in SE $\frac{1}{4}$ sec. 34, T. 2 S., R. 5 E., on right bank 3 miles upstream from Ross Creek and Hailstone. Section 1 is about 120 ft upstream from gage.

Drainage area.—233 sq mi.

Date of flood.—June 13, Oct. 7, 1952.

Gage height.—4.66 ft, 1.58 at gage; 5.66 ft, 2.14 ft at section 1.

Peak discharge.—1,200 cfs, 64.8 cfs.

Computed roughness coefficient.—Manning $n = 0.045; 0.073$.

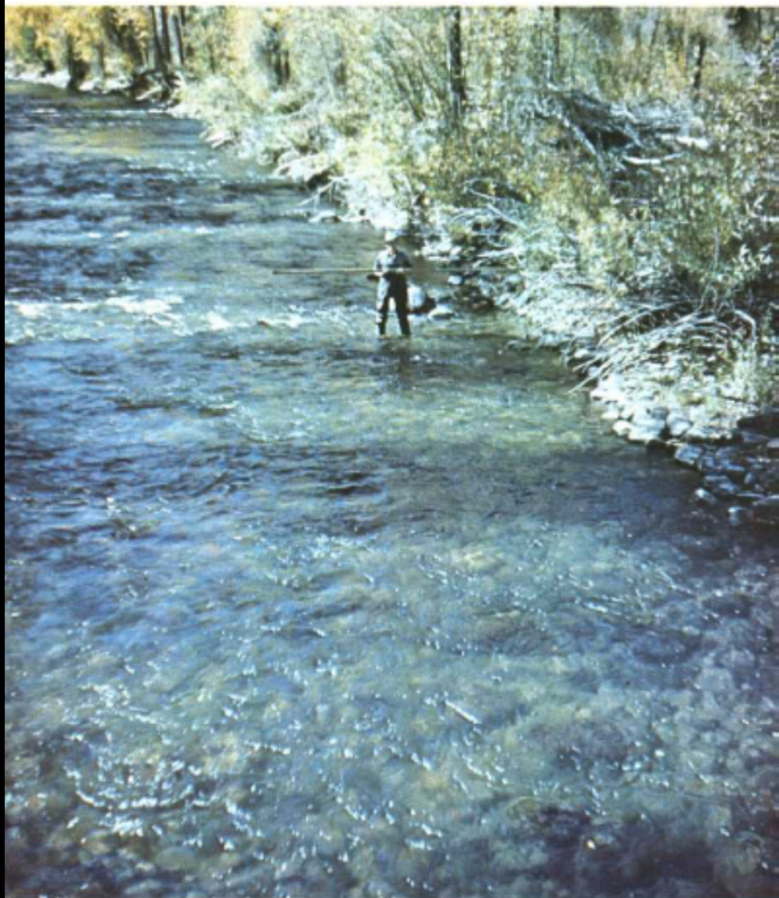
Description of channel.—Bed and banks consist of smooth rounded rocks as much as 1 ft in diameter. Some undergrowth is below water elevations of June 13.

Reach properties

Section	Area (sq ft)	Top width (ft)	Mean depth (ft)	Hydraulic radius (ft)	Mean velocity (ft per sec)	Length (ft) between sections	Fall (ft) between sections
June 13, 1952							
1.....	184	47	3.9	3.70	6.52
3.....	171	49	3.5	3.33	7.02	88	0.67
5.....	173	55	3.1	3.02	6.95	109	1.04
7.....	173	48	3.6	3.43	6.95	117	1.10
9.....	183	55	3.3	3.22	6.56	116	1.04
Oct. 7, 1952							
1.....	36	38	1.0	0.95	1.79
3.....	38	34	1.1	1.10	1.70	88	0.32
5.....	34	32	1.1	.82	1.90	109	.84
7.....	34	39	.9	.86	1.91	117	1.28
9.....	31	41	.8	.76	2.08	116	1.12

Notes.—

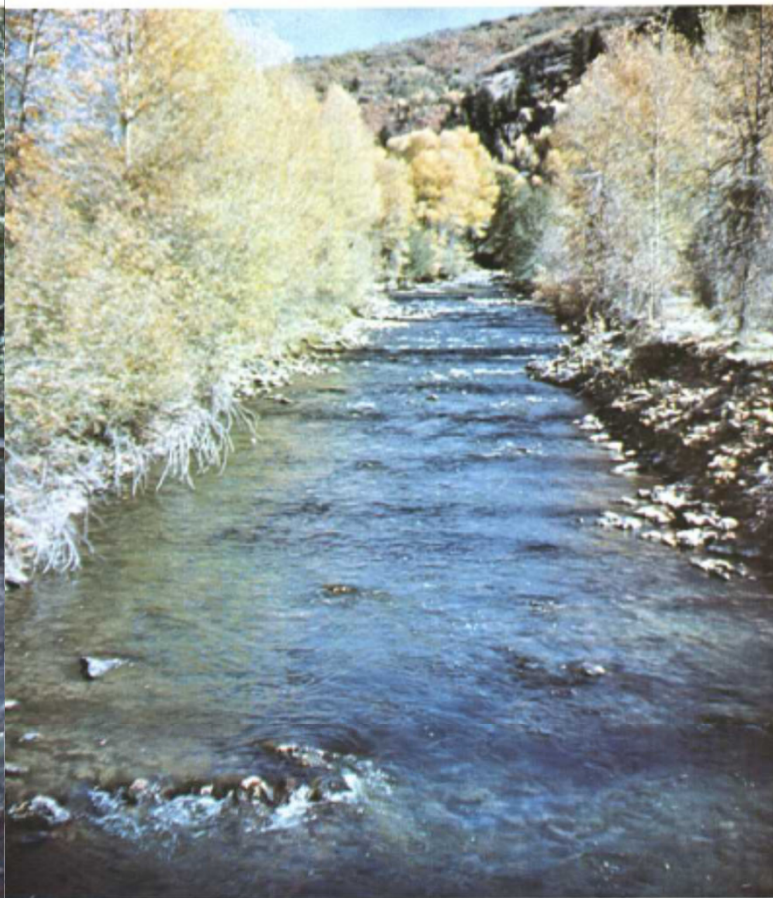
$n = 0.045; 0.073$



No. 769 downstream from section 3, Provo River near Hailstone, Utah.

136

$n = 0.045; 0.073$



No. 770 upstream from section 3, Provo River near Hailstone, Utah.

137



Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains

United States Geological Survey Water-supply Paper 2339

Metric Version

Authors: G.J. Arcement, Jr. and V.R. Schneider, USGS

Channel n Values

The most important factors that affect the selection of channel n values are:

1. the type and size of the materials that compose the bed and banks of the channel
2. the shape of the channel.

Cowan (1956) developed a procedure for estimating the effects of these factors to determine the value of n for a channel. The value of n may be computed by

$$n = (n_b + n_1 + n_2 + n_3 + n_4)m \quad (3)$$

where :

n_b = a base value of n for a straight, uniform, smooth channel in natural materials

n_1 = a correction factor for the effect of surface irregularities

n_2 = a value for variations in shape and size of the channel cross section,

n_3 = a value for obstructions

n_4 = a value for vegetation and flow conditions

m = a correction factor for meandering of the channel

Combining the tools

Relating Q to τ in a simple, prismatic, wide channel

This is how we link transport rate (a function of τ) to water discharge Q

$$Q = BhU$$

$$B = \alpha Q^\beta$$

$$h = \tau_o / \rho g S$$

$$U = \frac{\sqrt{S}}{n} h^{2/3} \text{ so}$$

$$Q = \left(\alpha Q^\beta \right) \left(\frac{\tau}{\rho g S} \right)^{5/3} \frac{\sqrt{S}}{n} \text{ or}$$

$$Q = \left[\frac{\alpha \sqrt{S}}{n} \left(\frac{\tau}{\rho g S} \right)^{5/3} \right]^{1/1-\beta}$$

Discharge = width * depth * velocity *continuity*

Width increases with discharge *channel geometry*

The 'depth-slope' product *momentum*

Manning's eqn *flow resistance*

Combine. It's only algebra!

Boundary stress is related to discharge, slope, roughness, channel shape

This solves the FLOW problem

Why?

$$Q = BhU$$

$$B = \alpha Q^\beta$$

$$h = \tau / \rho g S$$

$$U = \frac{\sqrt{S}}{n} h^{2/3} \text{ so}$$

$$Q = \left(\alpha Q^\beta \right) \left(\frac{\tau}{\rho g S} \right)^{5/3} \frac{\sqrt{S}}{n} \text{ or}$$

$$Q = \left[\frac{\alpha \sqrt{S}}{n} \left(\frac{\tau}{\rho g S} \right)^{5/3} \right]^{1/1-\beta}$$

$$Q_c = \left[\frac{\alpha \sqrt{S}}{n} \left(\frac{\tau_c}{\rho g S} \right)^{5/3} \right]^{1/1-\beta}$$

Because we generally relate changes in the bed and banks – **erosion and deposition** – to the bed shear stress τ_o and we will want to be able to relate τ_o to the water discharge e.g. at what Q will τ_o be sufficient to move river bed material?

For example, if the grain size of the river bed is used to find the critical shear stress for incipient motion τ_c , this relation gives us the critical discharge Q_c needed to move the river bed.

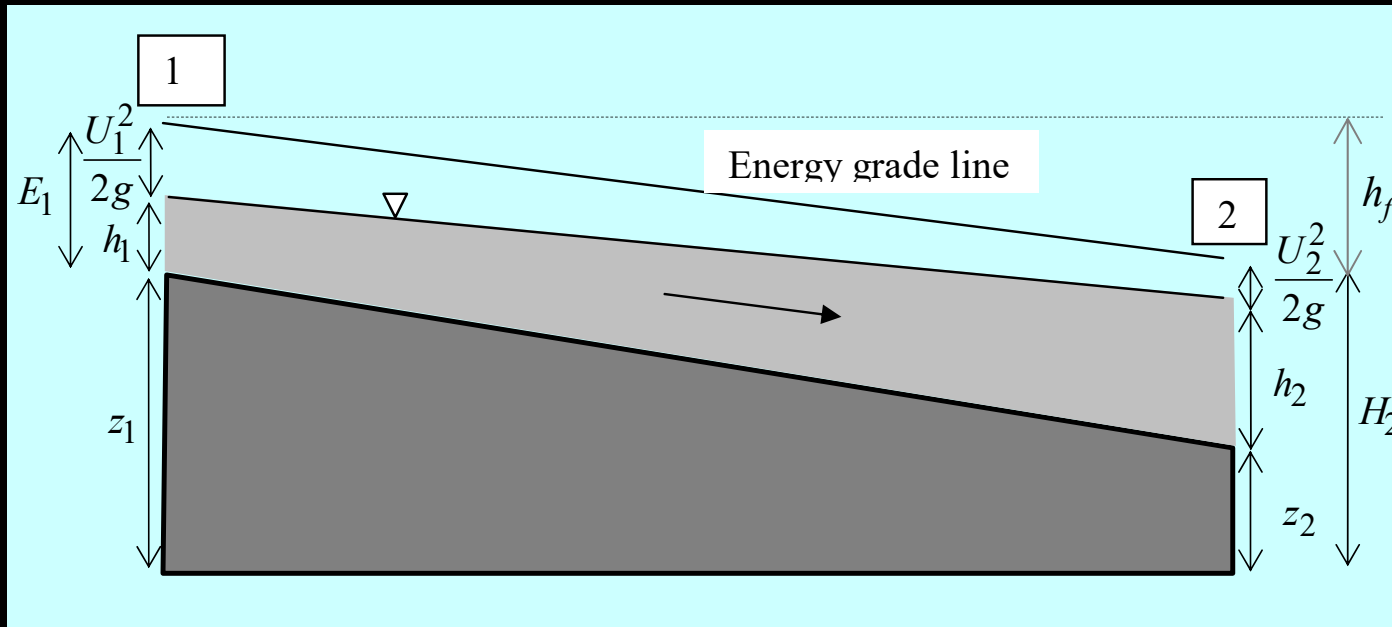
Conservation of Energy

- Units: Length
- Head 'loss'
- H, EGL
- Components

$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f$$

or

$$H_1 = H_2 + h_f$$



- **Gradually varied flow**

Estimate h_f use energy eqn. to estimate depth at one place given depth at another place (next)

- **Rapidly varied flow**

- (gates, weirs, rapid changes in width or bottom elevation)
- flow often complex, but can be approximated by assuming negligible energy loss (*if flow converging*)
- Over short distances, with small energy loss,

$z_1 \approx z_2$ and $h_f \approx 0$, so

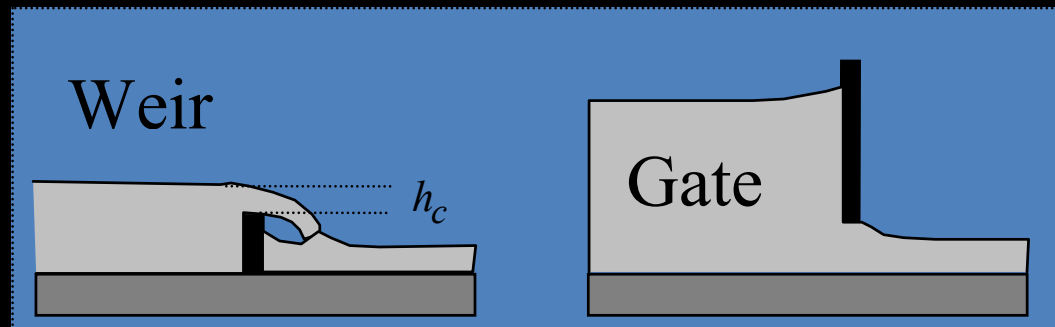
$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f$$

becomes $h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g}$

or

$$E_1 = E_2$$

$$E = h + \frac{U^2}{2g}$$



The Specific Energy E

flow energy relative to the bed

$$E = h + \frac{U^2}{2g}$$

$Q = UA$ For a rect. channel

$Q = Ubh$ Define

$$q \equiv Q/b = Uh$$

so $U = q/h$ and

$$E = h + \frac{q^2}{2gh^2}$$

The extraordinary properties of the Specific Energy E

$$E = h + \frac{q^2}{2gh^2}$$

- 2 flow states
- Change in h with step or constriction
- Properties of $\text{Min}(E)$

$$F = 1$$

$$F = \frac{U}{\sqrt{gh}} = \frac{q}{\sqrt{gh^3}} = 1$$

To find the minimum E , set $\frac{dE}{dh} = 0$.

$$\frac{d}{dh} \left(h + \frac{q^2}{2gh^2} \right) = 0$$

$$1 - \frac{q^2}{gh^3} = 0$$

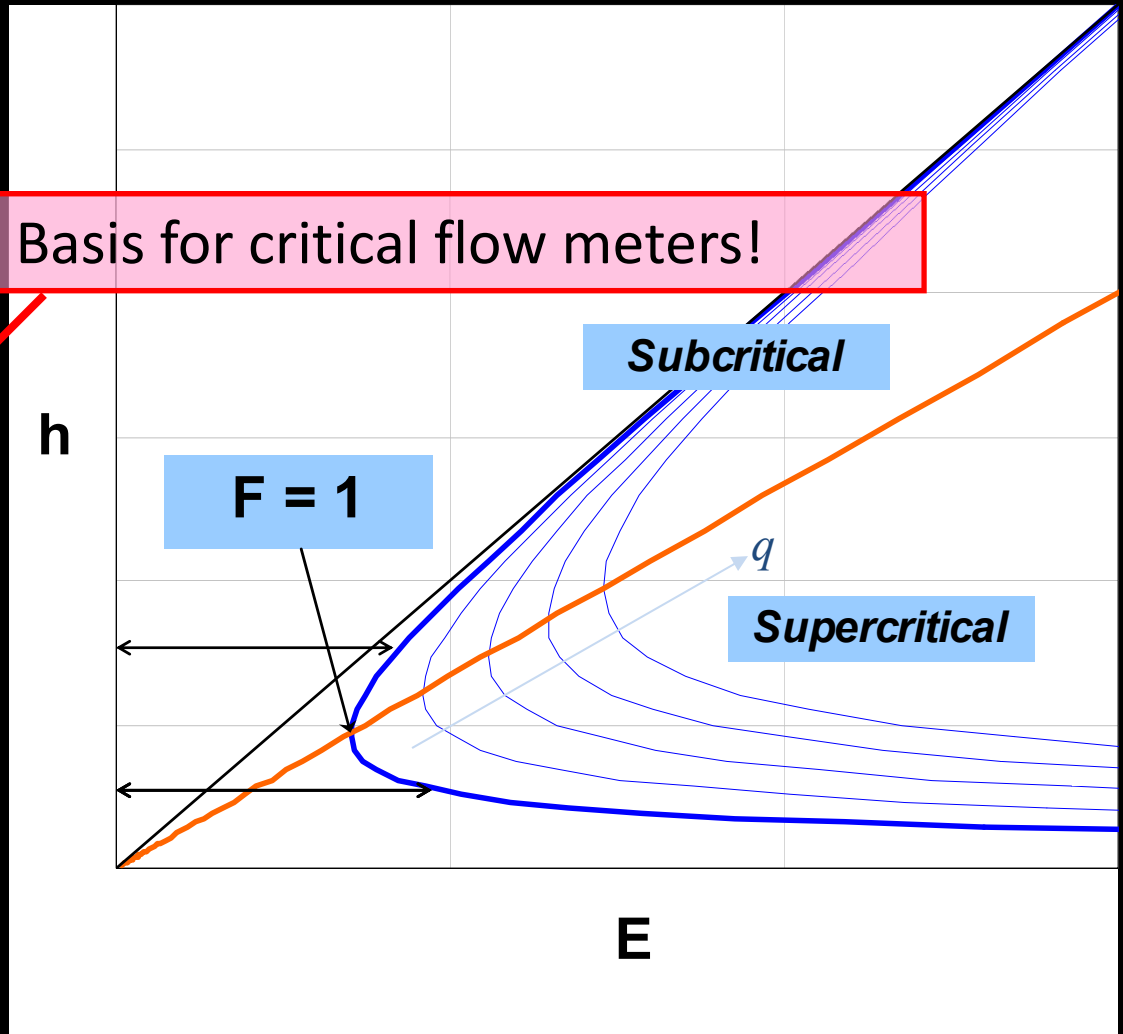
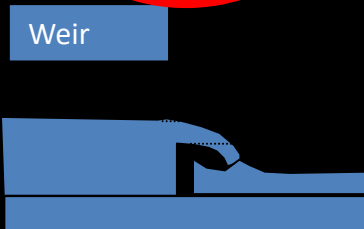
$$\frac{q^2}{gh^3} = \frac{U^2}{gh} = 1 = F^2$$

so $F = 1$ at minimum E .

Note, too, that $q = \sqrt{gh_c^3}$

where a subscript 'c' is included

to indicate that this holds at critical flow.



What happens when flow goes over a smooth step of height Δz , or through a contraction in width, with negligible head loss (H const)?

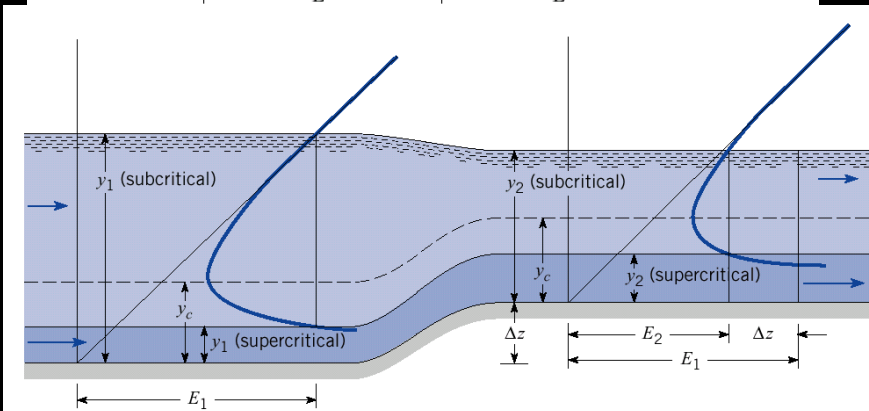
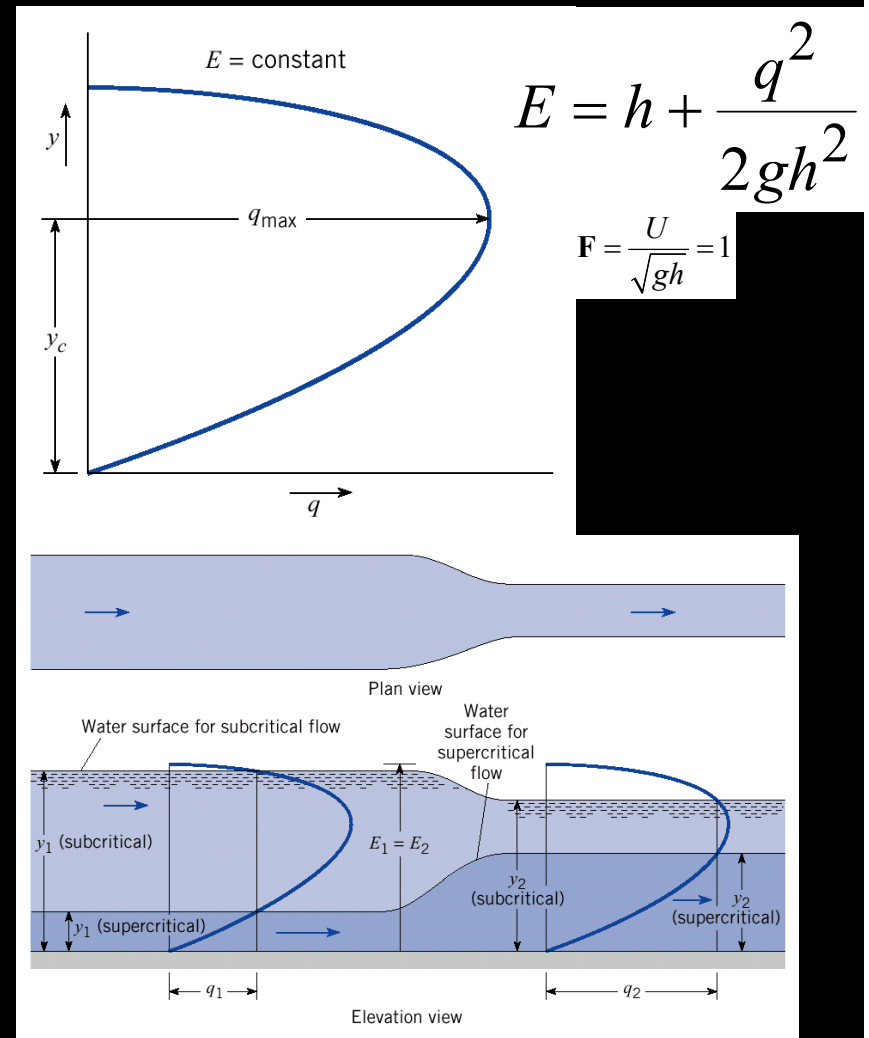
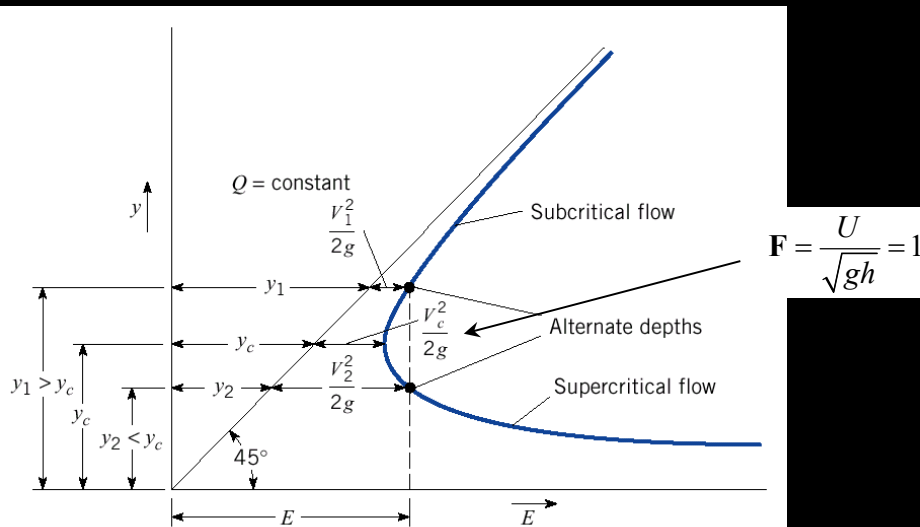
Flow Over Step

$$z_1 + \Delta z = z_2 \text{ and } h_f \approx 0,$$

$$z_1 + E_1 = z_2 + E_2$$

or $E_1 - \Delta z = E_2$

Flow Through Contraction $E_1 = E_2$



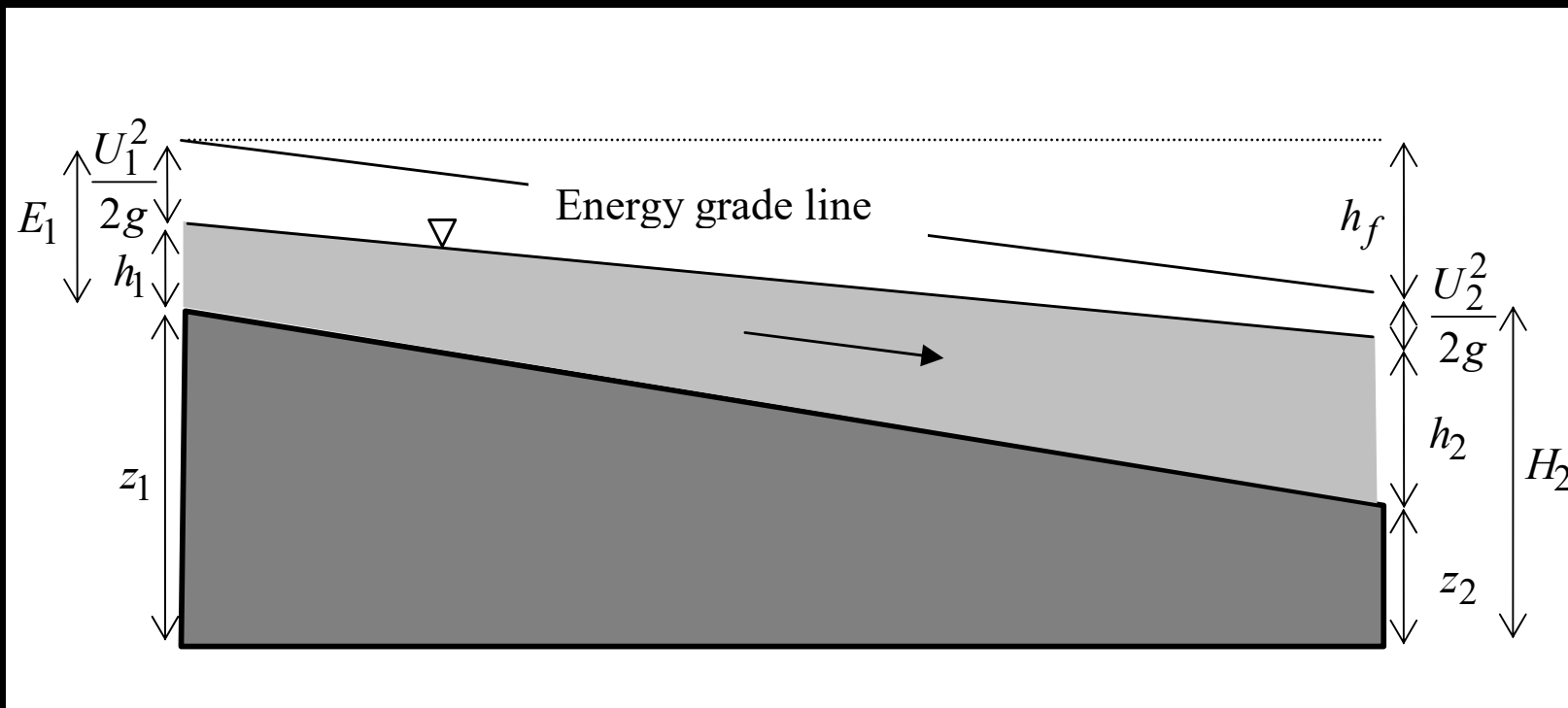
Diagrams from Roberson & Crowe, Fluid Mechanics, Wiley
Diagrams use y for flow depth and V for velocity

Gradually Varied Flow: Modeling Water Surface Profiles aka backwater or step-backwater modeling

Switching back depth from y to h and mean velocity from V to U Sorry!

Conservation of energy in open channel flow

$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f$$



Long distances: $Z_1 > Z_2$ and $h_f > 0$

The rate of head loss is defined as S_f

$$S_f = \frac{h_f}{\Delta x}$$

Must estimate head loss, so we need

length of stream over which the head loss is occurring

$$\Delta x = (x_2 - x_1)$$

$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f$$

$$\left(z_2 + h_2 + \frac{U_2^2}{2g} \right) - \left(z_1 + h_1 + \frac{U_1^2}{2g} \right) = -h_f = -S_f \Delta x$$

$$\frac{\left(z_2 + h_2 + \frac{U_2^2}{2g} \right) - \left(z_1 + h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = \frac{\Delta H}{\Delta x} \equiv -S_f$$

Just restating the definition of energy in open channel flow

Using

$$\frac{z_2 - z_1}{x_2 - x_1} \equiv -S_o$$

$$\frac{\left(z_2 + h_2 + \frac{U_2^2}{2g} \right) - \left(z_1 + h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = \frac{\Delta H}{\Delta x} \equiv -S_f$$

becomes

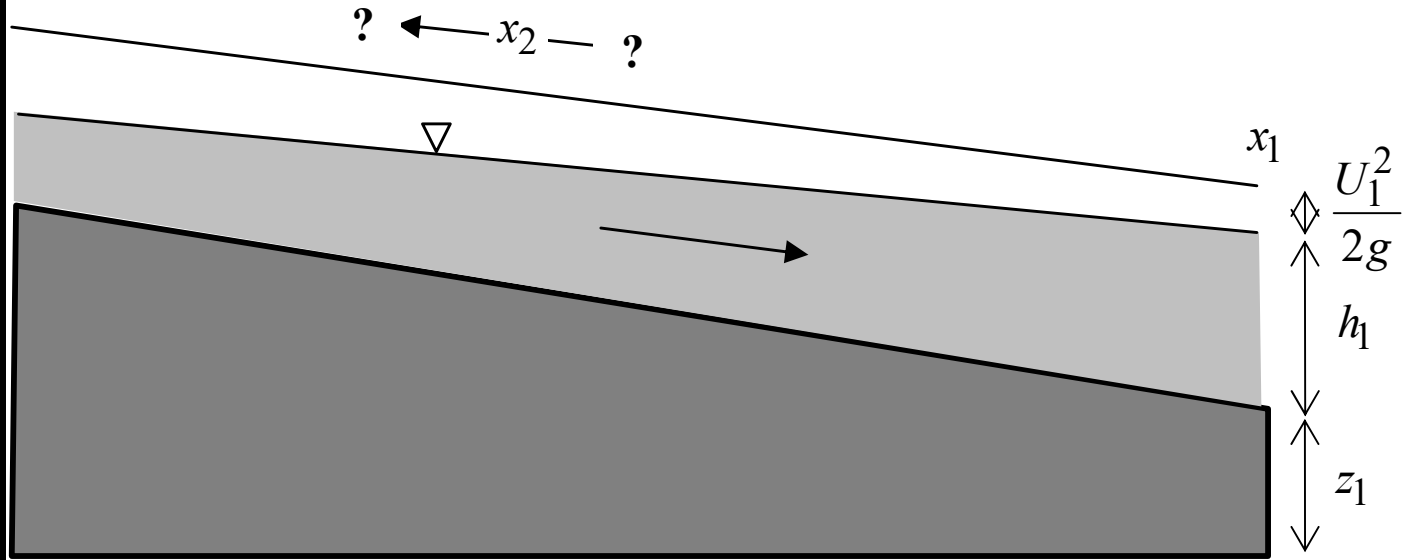
$$\frac{\left(h_2 + \frac{U_2^2}{2g} \right) - \left(h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = S_o - S_f$$

An ordinary differential equation that tells us the *rate* ($\Delta h/\Delta x$) at which h changes alongstream.

This relation only helps if we know the depth at some place to start with. If we do, then the equation can be used to calculate depth at some other location and we are off to the races.

Given h_1 at x_1 , at what x_2 will h_2 occur?

Direct Step



$$\frac{\left(h_2 + \frac{U_2^2}{2g} \right) - \left(h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = S_o - S_f$$

$$x_2 = x_1 + \frac{\left(h_2 + \frac{U_2^2}{2g} \right) - \left(h_1 + \frac{U_1^2}{2g} \right)}{S_o - S_f}$$

Specify Q , S_o , and h_1 .

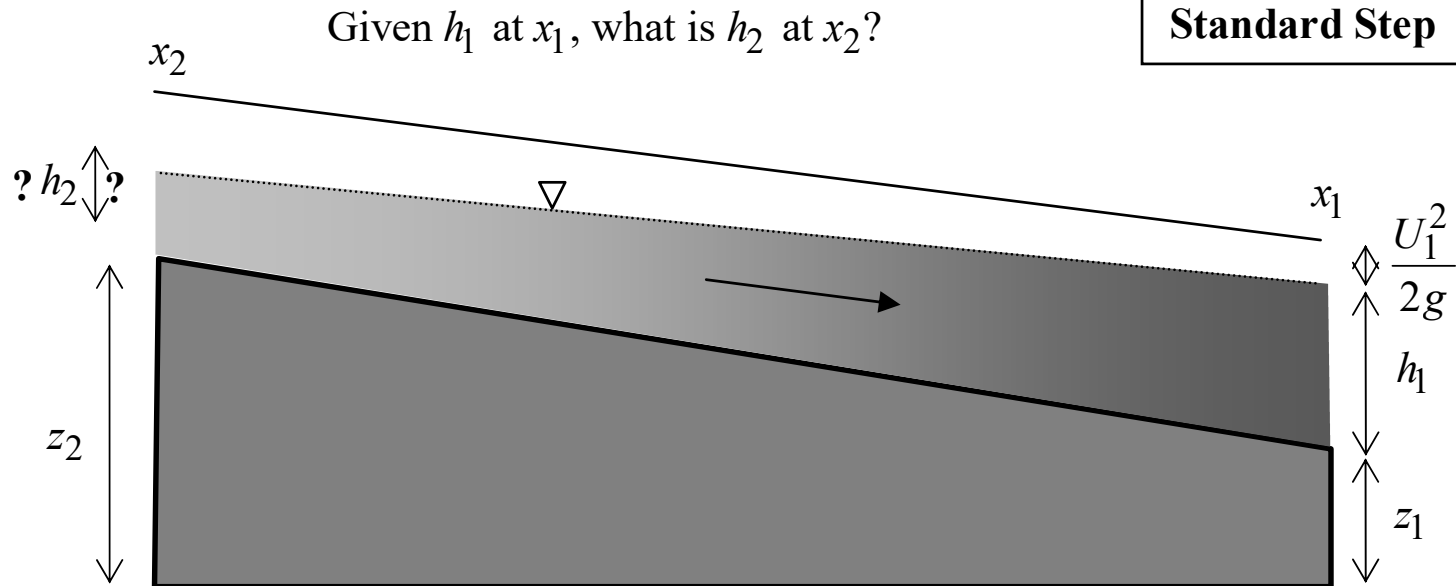
Continuity gives U_1

Pick h_2 , continuity gives U_2

Solve for x_2 .

Not very convenient, except for prismatic canals!

Standard Step



$$\frac{\left(h_2 + \frac{U_2^2}{2g} \right) - \left(h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = S_o - S_f$$

$$h_2 = h_1 + \frac{1}{2g} \left(U_1^2 - U_2^2 \right) (S_o - S_f) (x_2 - x_1)$$

Specify Q , S_o , h_1 and Δx

Continuity gives U_1

Guess h_2 , continuity gives U_2

Iterate.

Defining S_f . We used Manning's eqn to describe flow resistance

$$U = \frac{\sqrt{S}}{n} R^{2/3}$$

For steady, uniform flow, $S_o = S_f$ and we did not have to specify which S was used in Manning's eq.

$$S_f = \left(\frac{nU}{R^{2/3}} \right)^2$$

We now precisely define the slope in Mannings's eqn as S_f and we use Manning's eq. to define S_f .

$$h_2 = h_1 + \frac{1}{2g} (U_1^2 - U_2^2) (S_o - S_f) (x_2 - x_1)$$

Having chosen a value of h_2 , we calculate R_2 , U_2 , and S_f . Inserting U_2 and S_f on the right side, we then calculate a new, improved guess for h_2 .

This is step-backwater modeling. This is HEC-RAS.

Big advantages of a canned program:

- (a) Iterative solution to the governing equation is not always stable
- (b) The world is messy (irregular cross-sections, variable roughness, compound channels, bridges, culverts,)

Don't believe a RAS solution unless you can sketch it yourself! How?

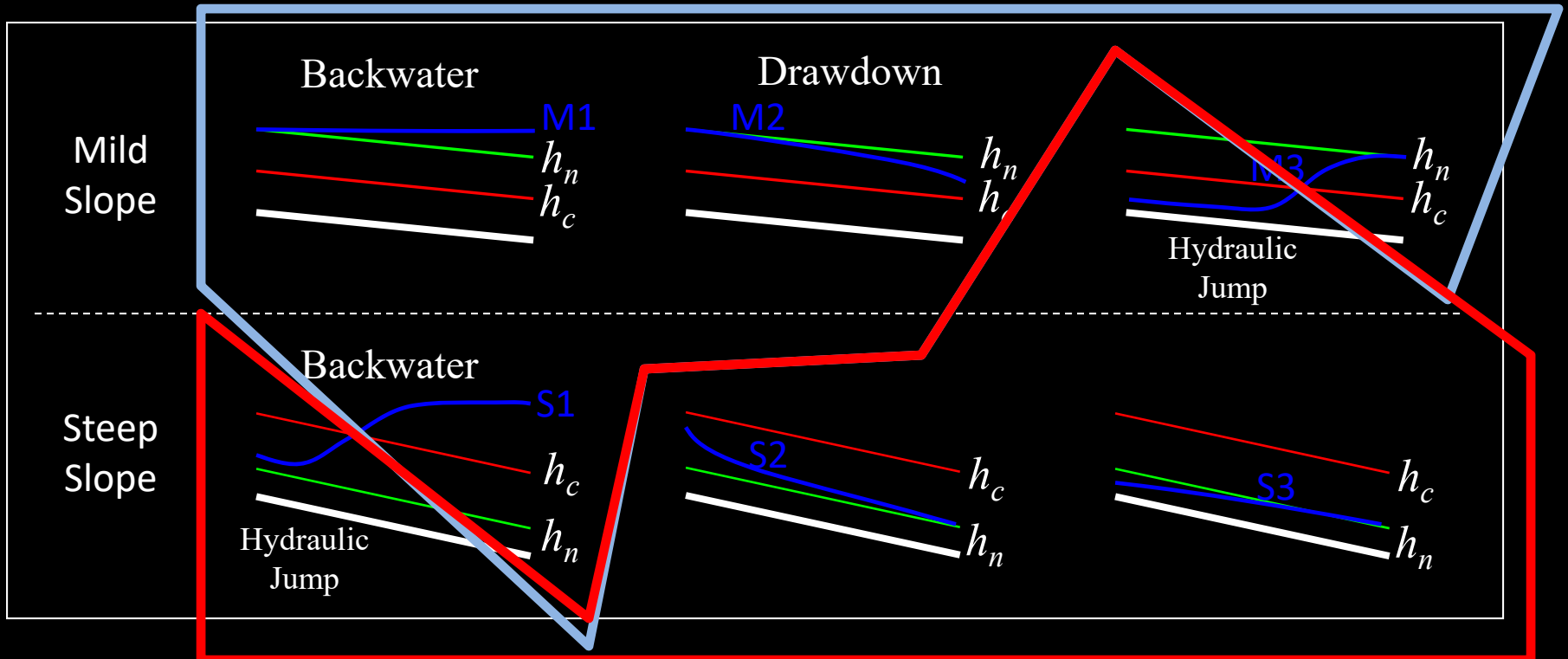
- (1) identify controls (known depth for a specified discharge)
critical depth, normal flow
- (2) which way to go (start D/S in subcritical flow and U/S in supercritical flow)
- (3) Sketch!

In its most basic application, to build a HEC-RAS model you (a) survey cross sections in your study reach and enter these into RAS, (b) assign roughness to each section (or portions of sections), (c) specify a discharge, and (d) step back and let it run.

ROUGHNESS: If you don't calibrate, you are only guessing.

Types of gradually varied flow profiles

Subcritical Flow – Downstream Control!



Supercritical Flow – Upstream Control!

$$h_c = \left(q^2 / g \right)^{1/3}$$

aka $F = 1$

$$h_n = \left(\frac{nq}{\sqrt{S}} \right)^{3/5}$$

aka Manning's eq.

Some Key Points

How often does sediment move?

Flow **COMPETENCE** of bed material

Threshold Channel, Incipient motion

Shields curve + flood frequency

What is the Sediment Balance?

Sediment Supply v. Transport **CAPACITY**

Alluvial channel problem

Transport relations give capacity. Supply?
+ flow duration

All sediment transport problems have a FLOW PART

Flow problem → hydraulics → boundary stress τ_o

Sediment problem → sediment response to boundary stress τ_o

All relations are approximate. You can quantitatively evaluate the error associated with approximations (e.g. $\tau_o = \rho g R S$)

Channel hydraulics and transport: uncertainty is in the input!

can estimate the magnitude and importance of uncertainty

need actual measurements to obtain accuracy

The tools are general, their application is local.

RAS (*et al.*) calculates sediment transport. Plug and Play?

Provides useful basis for estimating flow and stress

Accounts (coarsely) for nonuniform flow conditions

RAS is very good at what it is meant to do: calculate water surface elevation. For sediment transport: *it's so easy to be so wrong*

(1a) Finding normal depth in a rectangular channel

(1a) Finding Normal Depth - Rectangular Channel

$$V = \frac{R_h^{2/3}}{n} \sqrt{S_o} \quad \text{and} \quad Q = \frac{A^{5/3}}{nP^{2/3}} \sqrt{S_o} \quad \oplus$$

where

$$A = bh, \quad P = b + 2h, \quad R_h = \frac{bh}{b + 2h}$$

Given $R_h, n, Q \Rightarrow$ Find S_o from \oplus

Given $R_h, n, S_o \Rightarrow$ Find Q from \oplus

Given $R_h, Q, S_o \Rightarrow$ Find n from \oplus

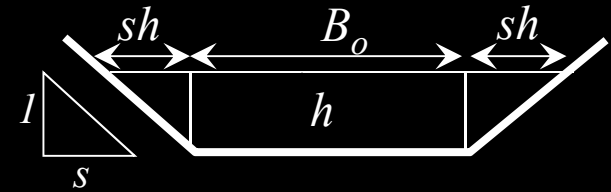
Given $S_o, n, Q \Rightarrow$ To find h , have to iterate!

Try \oplus arranged as
$$Q = \frac{b}{n} \frac{h^{5/3}}{\left(1 + 2\frac{h}{b}\right)^{2/3}} \sqrt{S_o}$$

Solve for y on top:
$$h_{n+1} = \left(\frac{nQ}{b\sqrt{S_o}}\right)^{3/5} \left(1 + 2\frac{h_n}{b}\right)^{2/5}$$

where n and $n + 1$ subscripts indicate successive approximations

(1b) Finding normal depth in a trapezoidal channel



(1b) Finding Normal Depth - Trapezoidal Channel

$$\text{Manning's eqn: } Q = \frac{\sqrt{S_o}}{n} \frac{[h(B_o + sh)]^{5/3}}{\left[B_o + 2h\sqrt{1+s^2} \right]^{2/3}} \quad \otimes$$

where

$$B = B_o + 2sh \quad A = B_o h + sh^2 = h(B_o + sh) \quad P = B_o + 2h\sqrt{1+s^2}$$

Given $h, n, Q \Rightarrow$ Find S_o from \otimes

Given $h, n, S_o \Rightarrow$ Find Q from \otimes

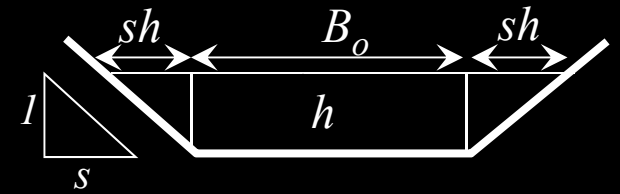
Given $h, S_o, Q \Rightarrow$ Find n from \otimes

Given $S_o, n, Q \Rightarrow$ To find h , have to iterate!

$$\text{Try } \otimes \text{ arranged as } h_{n+1} = \left(\frac{nQ}{\sqrt{S_o}} \right)^{3/5} \frac{\left[B_o + 2h_n \sqrt{1+s^2} \right]^{2/5}}{B_o + sh_n}$$

where n and $n + 1$ subscripts indicate successive approximations

(2) Finding critical depth in a rectangular or trapezoidal channel



(2) Finding critical depth

$$\text{Finding Critical Depth } F = 1 = \frac{V}{\sqrt{gh}} = \frac{Q/A}{\sqrt{g(A/B)}} \quad \text{or} \quad \frac{Q}{\sqrt{g}} = \frac{A^{3/2}}{\sqrt{B}}$$

where B is top width.

$$(2a) \text{ Rectangular Channel } \frac{Q}{\sqrt{g}} = \frac{A^{3/2}}{\sqrt{B}} = \frac{(Bh)^{3/2}}{\sqrt{B}} \quad \text{or} \quad \bar{h}_c = \left(\frac{Q}{\sqrt{gB}} \right)^{2/3}$$

(2b) Trapezoidal Channel

$$B = B_o + 2sh \quad A = B_o h + sh^2 = h(B_o + sh) \quad \frac{A^{3/2}}{\sqrt{B}} = \sqrt{\frac{h^3 (B_o + sh)^3}{B_o + 2sh}}$$

$$\frac{Q}{\sqrt{g}} = \sqrt{\frac{y^3 (B_o + sh)^3}{B_o + 2sh}}$$

$$\text{Given } h_c, B_o \quad \Rightarrow \quad \text{Find } Q = \sqrt{\frac{gh_c^3 (B_o + sh_c)^3}{B_o + 2sh_c}}$$

$$\text{Given } Q, B_o \quad \Rightarrow \quad \text{Find } y_c = \left(\frac{Q^2}{g} \frac{B_o + 2sh_c}{(B_o + sh_c)^3} \right)^{1/3}$$

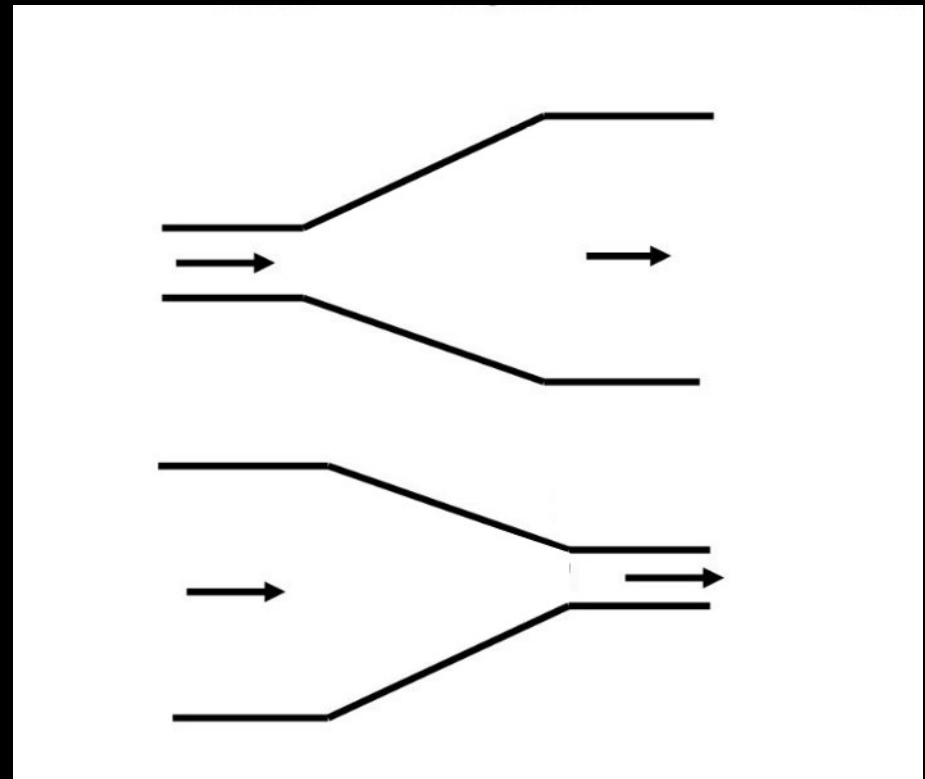
$$\text{Given } h_c, Q \quad \Rightarrow \quad \text{Find } B_o = \frac{gh_c^3}{Q^2} (B_o + sh_c)^3 - 2sh_c$$

When do I use momentum and when do I use energy?

They are both always true, but in most cases, one term in one of the two conservation relations is poorly known and, therefore, you don't have the information needed to solve the problem. Often, you can use the other relation to solve for velocity and then determine the unknown forces (momentum) or head losses (energy).

Expanding Flow: head losses are likely to be nonnegligible, whereas the flow force on the wall may be small. Thus, you use momentum assuming no wall force. After solving for the velocity in both sections, you can use energy to find the head loss.

Contracting Flow: flow force on the wall is likely to be nonnegligible, whereas the head losses may be small. Thus, you use energy assuming no head loss. After solving for the velocity in both sections, you can use momentum to find the wall force.



For standard channels and pipes, head loss has been carefully measured experimentally and head-loss coefficients are available to directly solve the energy problem.