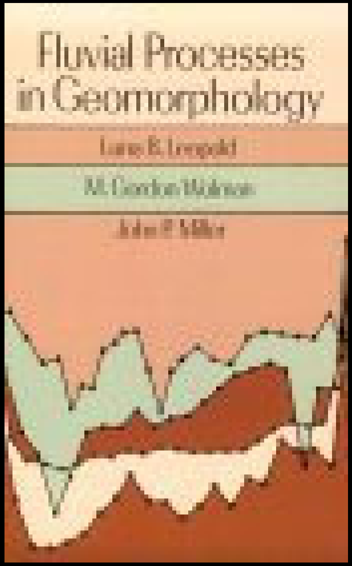
The Equilibrium Channel & Channel Change

Peter Wilcock
31 July 2024

The search for common empirical attributes of streams

Luna B. Leopold



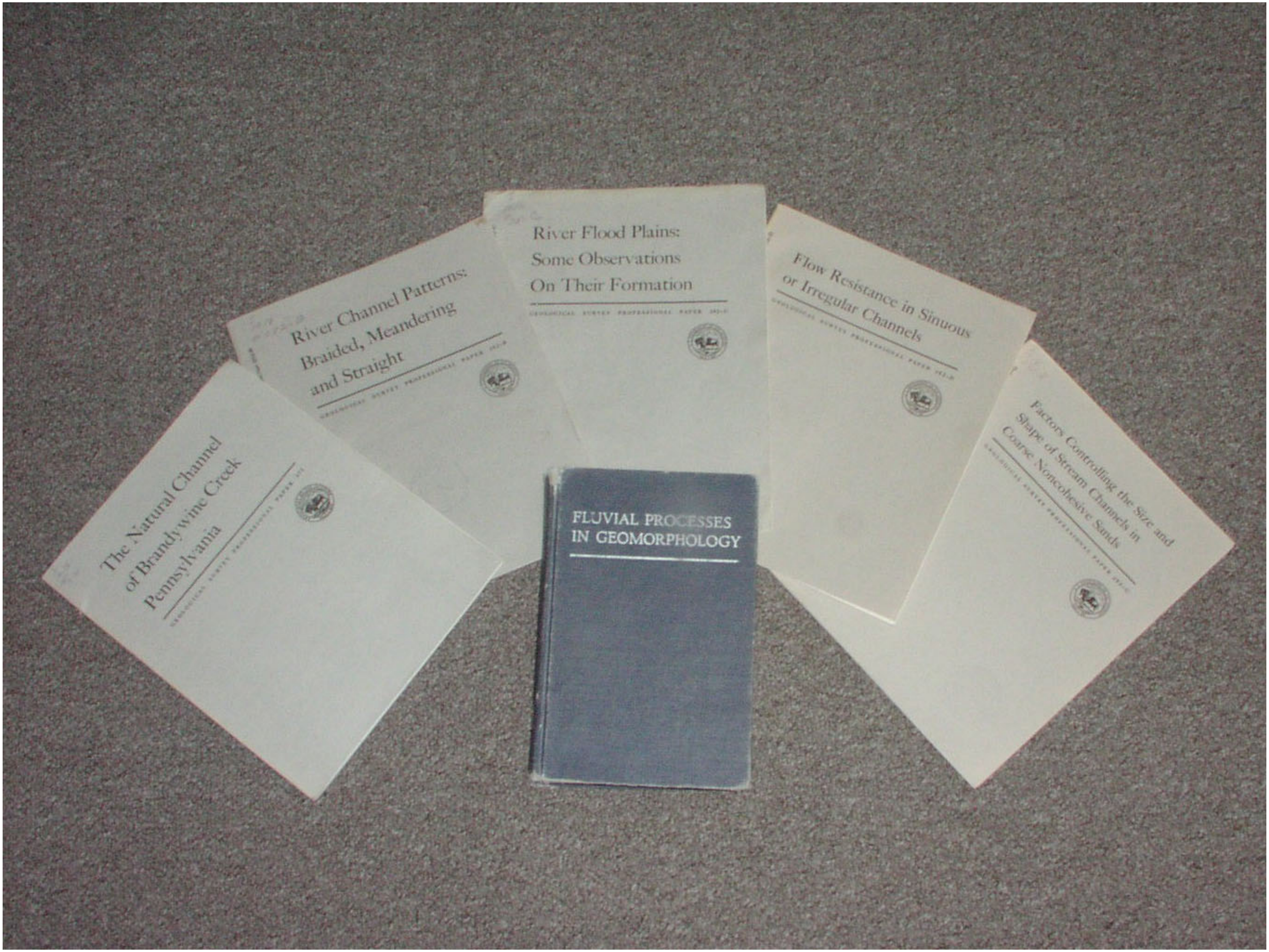
Fluvial Processes in Geomorphology

Luna B. Leopold

M. Gordon Wolman

John P. Miller

M. Gordon ("Reds") Wolman



The Natural Channel of Brandywine Creek Pennsylvania

River Channel Patterns: Braided, Meandering and Straight

River Flood Plains: Some Observations On Their Formation

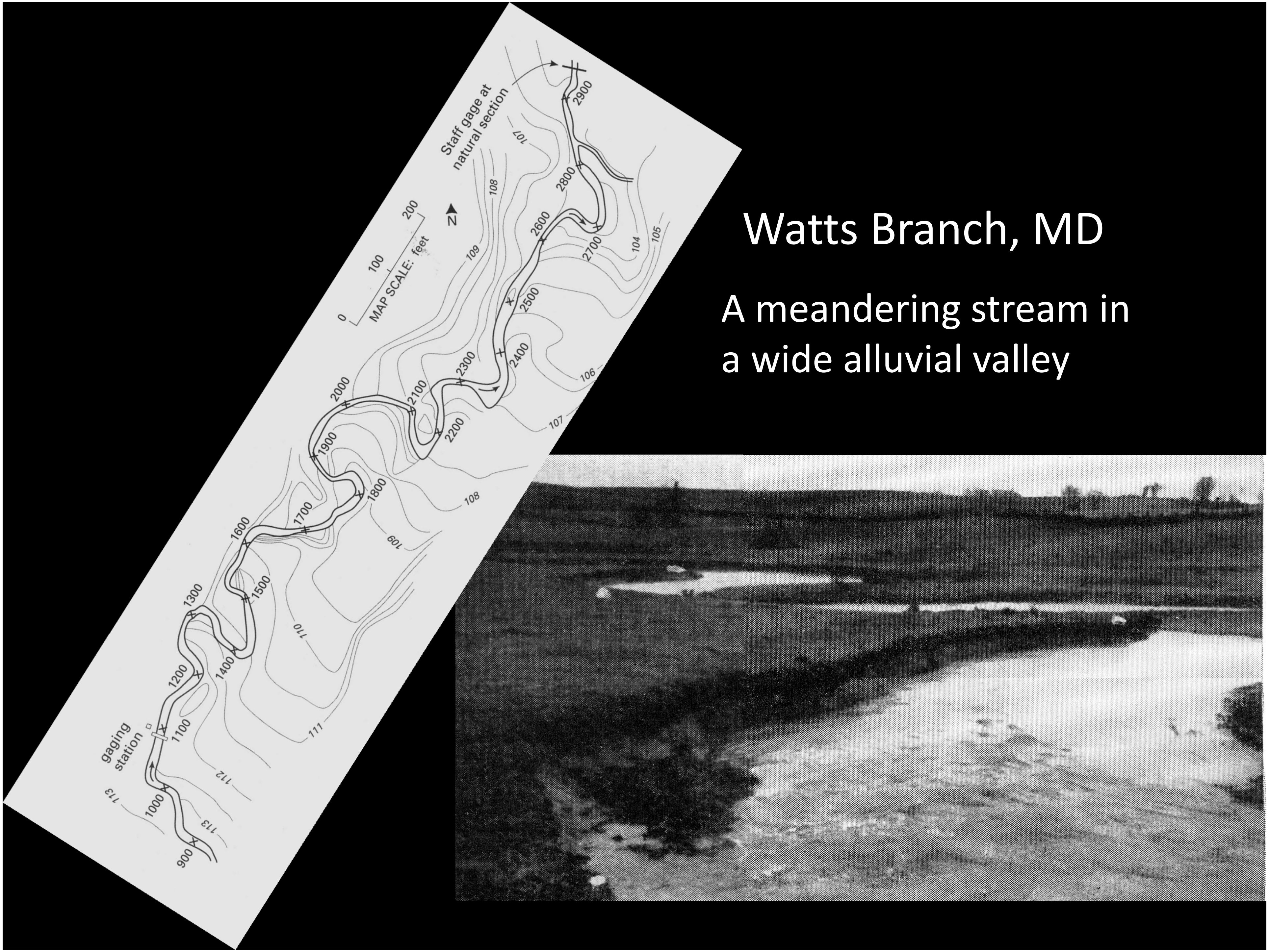
Flow Resistance in Sinuous or Irregular Channels

Factors Controlling the Size and Shape of Stream Channels in Coarse Noncohesive Sands

FLUVIAL PROCESSES IN GEOMORPHOLOGY

Watts Branch, MD

A meandering stream in a wide alluvial valley



The channel actively migrated across its valley, depositing a point bar on the inside of the meander bends and cutting a steep bank on the outside of the bends. The upper surface of the point bar was relatively flat.

The remarkable thing was that the channel appeared to maintain its size and shape as it migrated. It appeared to be in a state of ***dynamic equilibrium.***

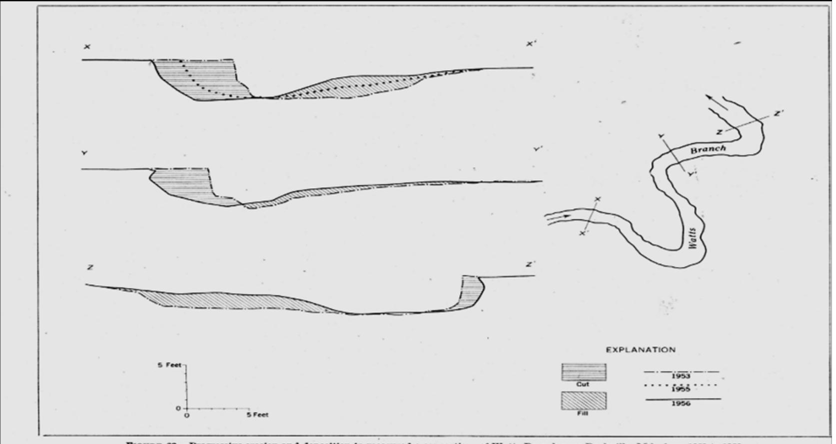
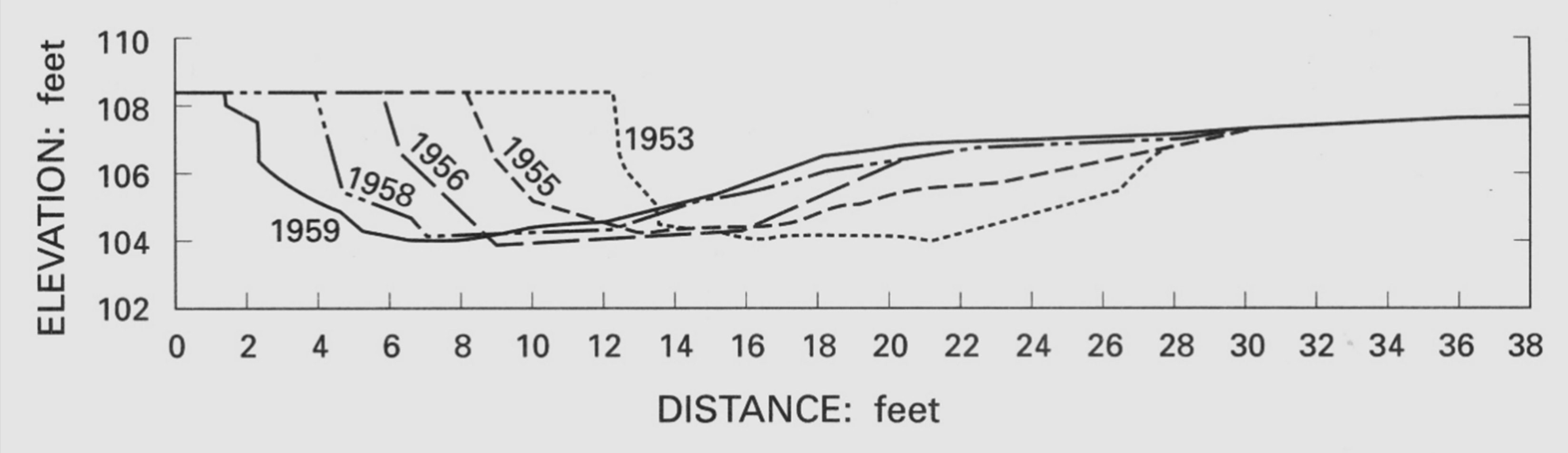
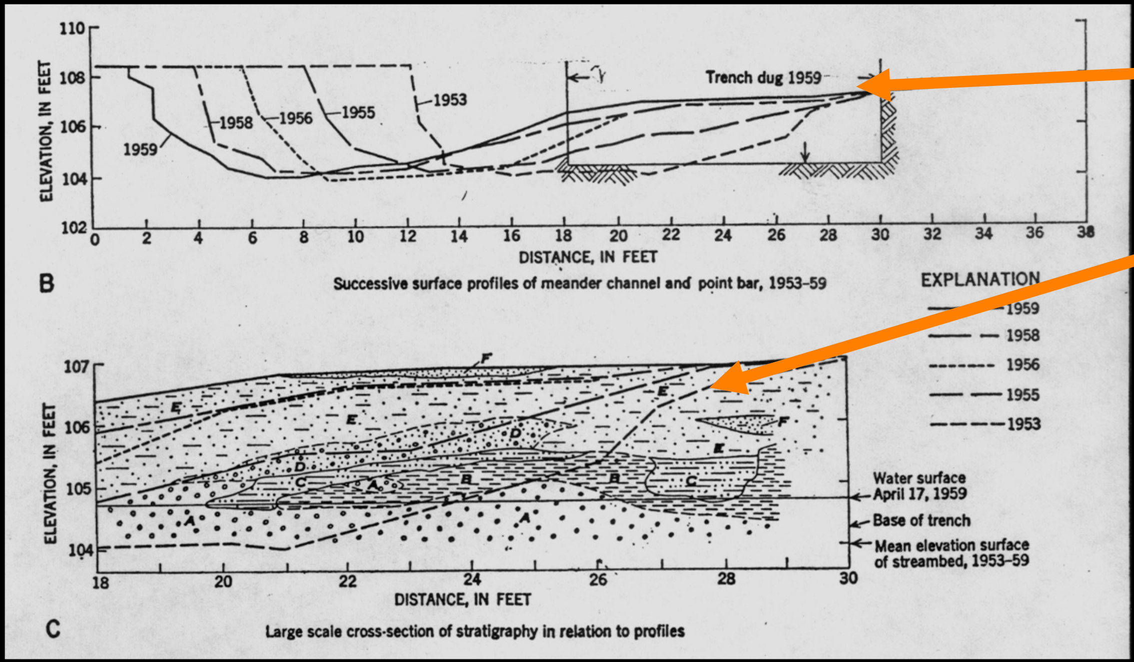


FIGURE 62.—Progressive erosion and deposition in measured cross sections of Watts Branch near Rockville, Md., from 1953 to 1956.





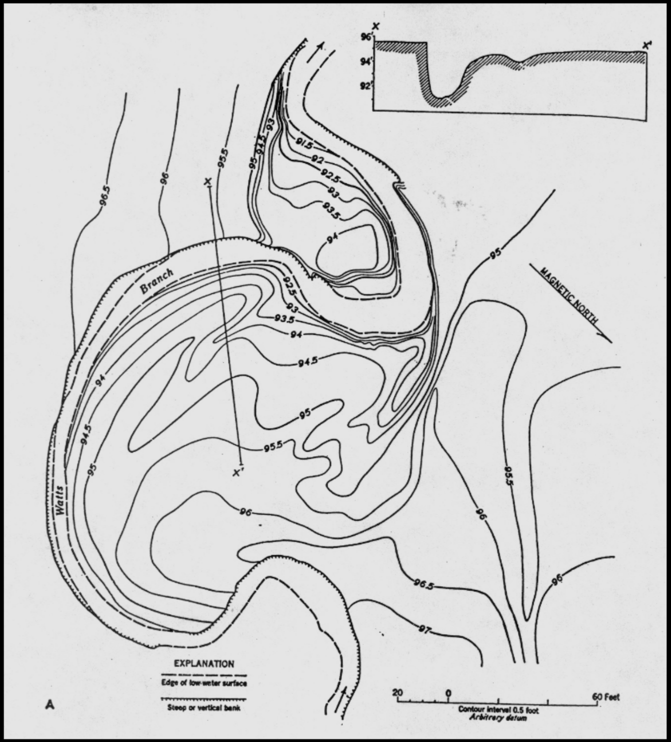
Successive surface profiles of meander channel and point bar, 1953-59

Large scale cross-section of stratigraphy in relation to profiles

Active floodplain

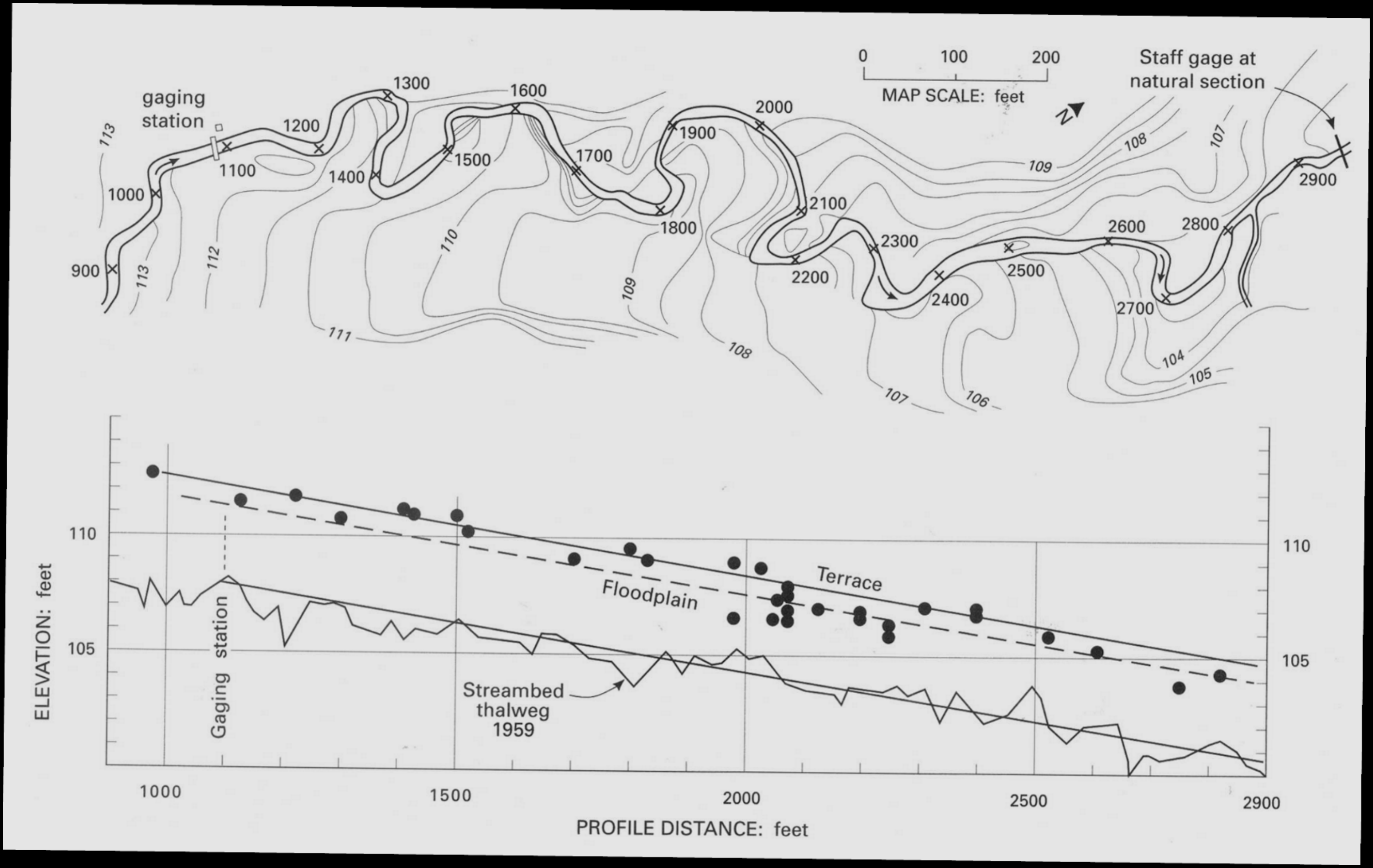
And its stratigraphy

The flat surface behind the cutbank is higher than the pointbar floodplain, presumably as the result of gradual overbank deposition during large floods over the period since it was originally deposited.



The architecture of the alluvial valley reflects a competition between lateral accretion and vertical accretion. At Watts Branch, lateral accretion was seen as dominant.

The active floodplain is being constructed in the present hydrologic regime. This landform is inundated by frequent floods. The elevation of flow that just begins to inundate this feature was defined as the bankfull stage.



The elevations of the active floodplain have a reach-average slope that is approximately parallel to that of the bed. Terraces are higher than the active floodplain with elevations that are parallel to the bed as well.

Bankfull Discharge

- The discharge that “just fills the channel to the top of the banks”.
- According to original analyses of Leopold et al. (1964), bankfull discharge reoccurs on average about every 1-2 years, although with a wide variability.
- Building on the work of Wolman and Miller (1960), they suggest that bankfull discharge and effective discharge are equivalent.

Effective Discharge? The flow that moves the most sediment over time.

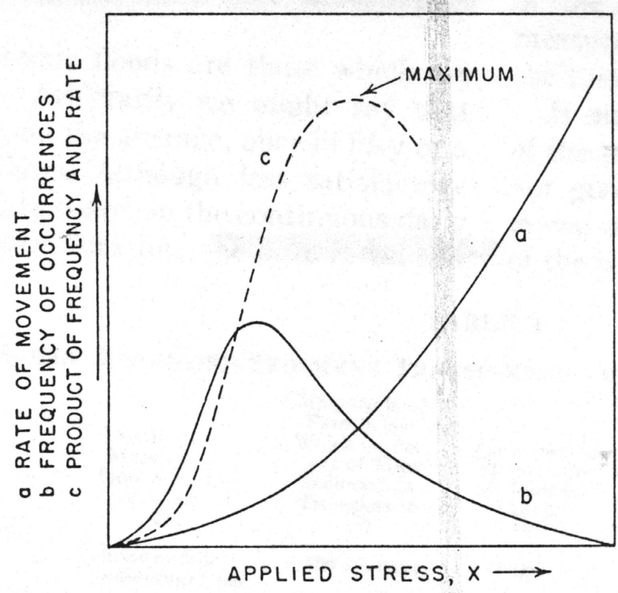


FIG. 1.—Relations between rate of transport, applied stress, and frequency of stress application.

Wolman & Miller (1960)

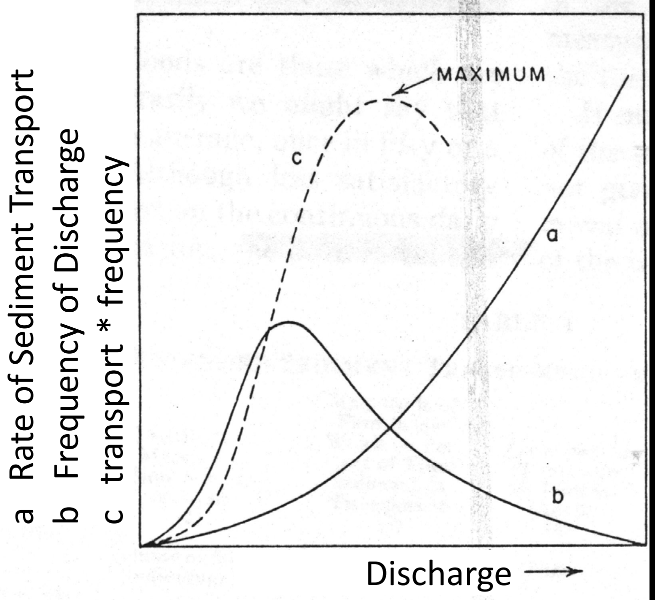


FIG. 1.—Relations between rate of transport, applied stress, and frequency of stress application.

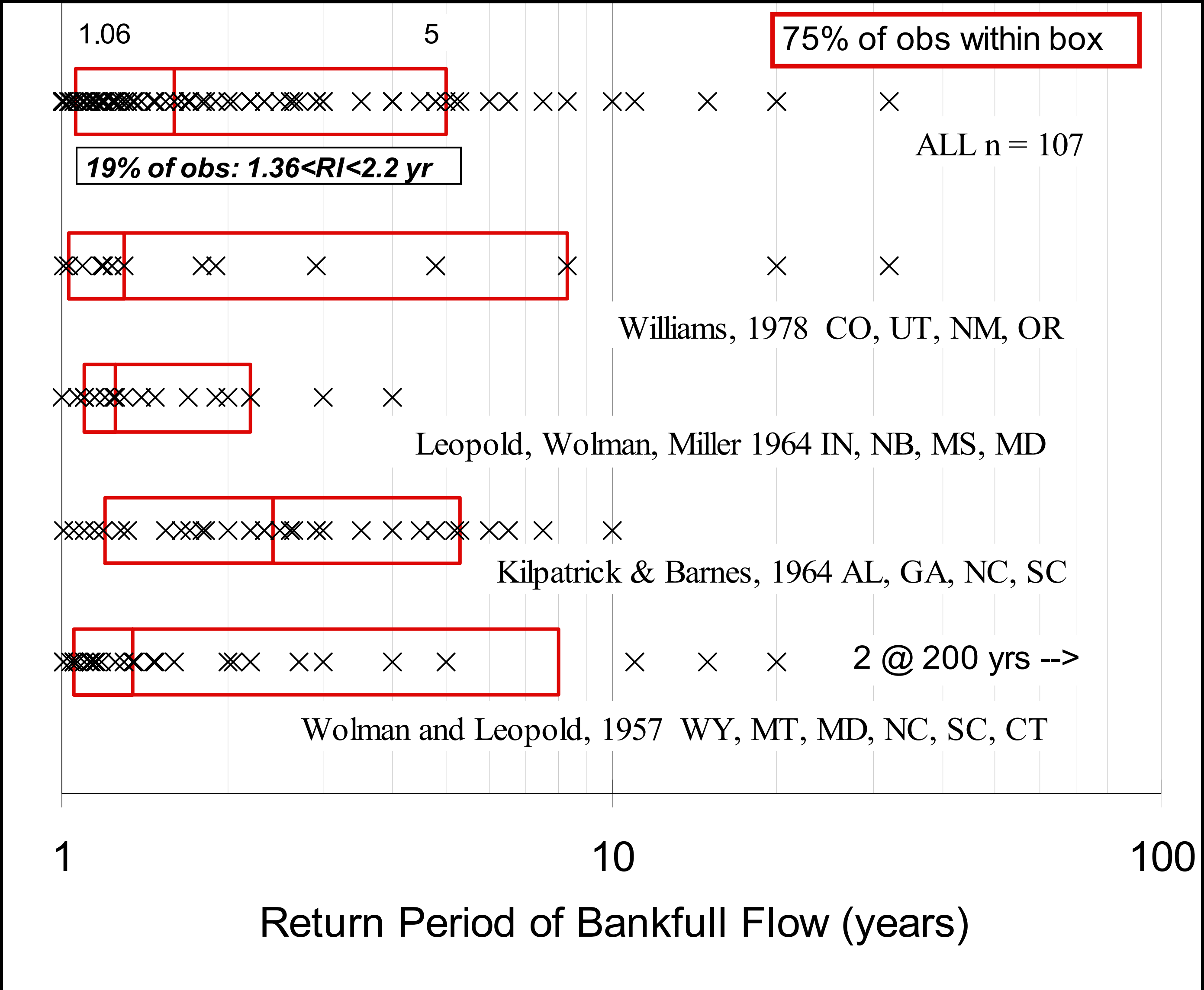
a Rate of Sediment Transport (tons / day)
b Frequency of Discharge (days / yr)
c transport * frequency (tons / yr)

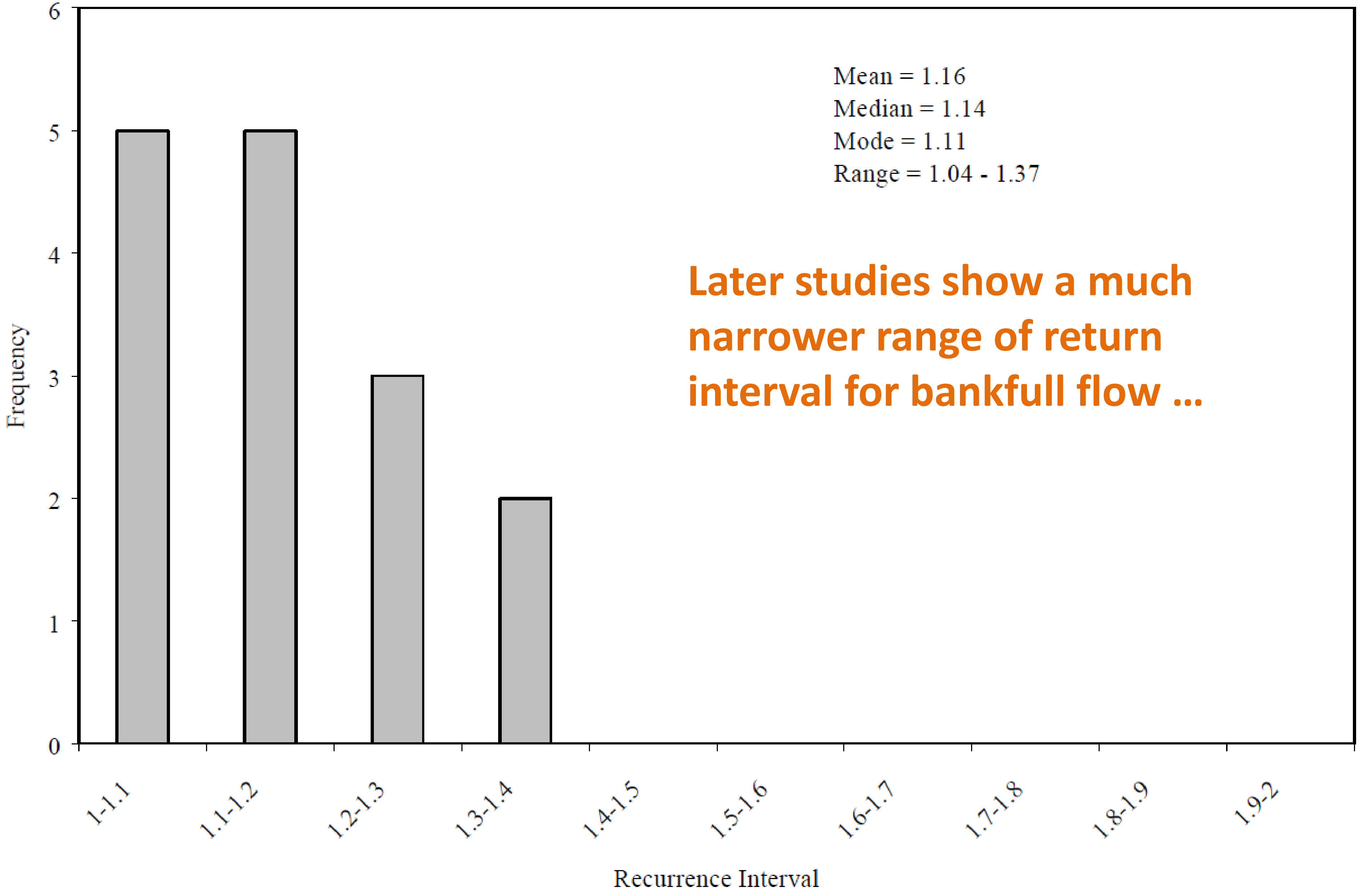
Curve c tells you how much sediment is moved by each flow. The area under the curve tells you how much sediment is moved each year.
Does the ***maximum*** of the curve tell us something specific about channel geometry?

Hypothesis:

Alluvial channels adjust their morphology to move the most sediment (over time) at the bankfull stage, which has a return period between 1-2 years

Some early studies on the magnitude of bankfull discharge show considerable variability, but a median in the 1-2 yr range …





Mean = 1.16
Median = 1.14
Mode = 1.11
Range = 1.04 - 1.37

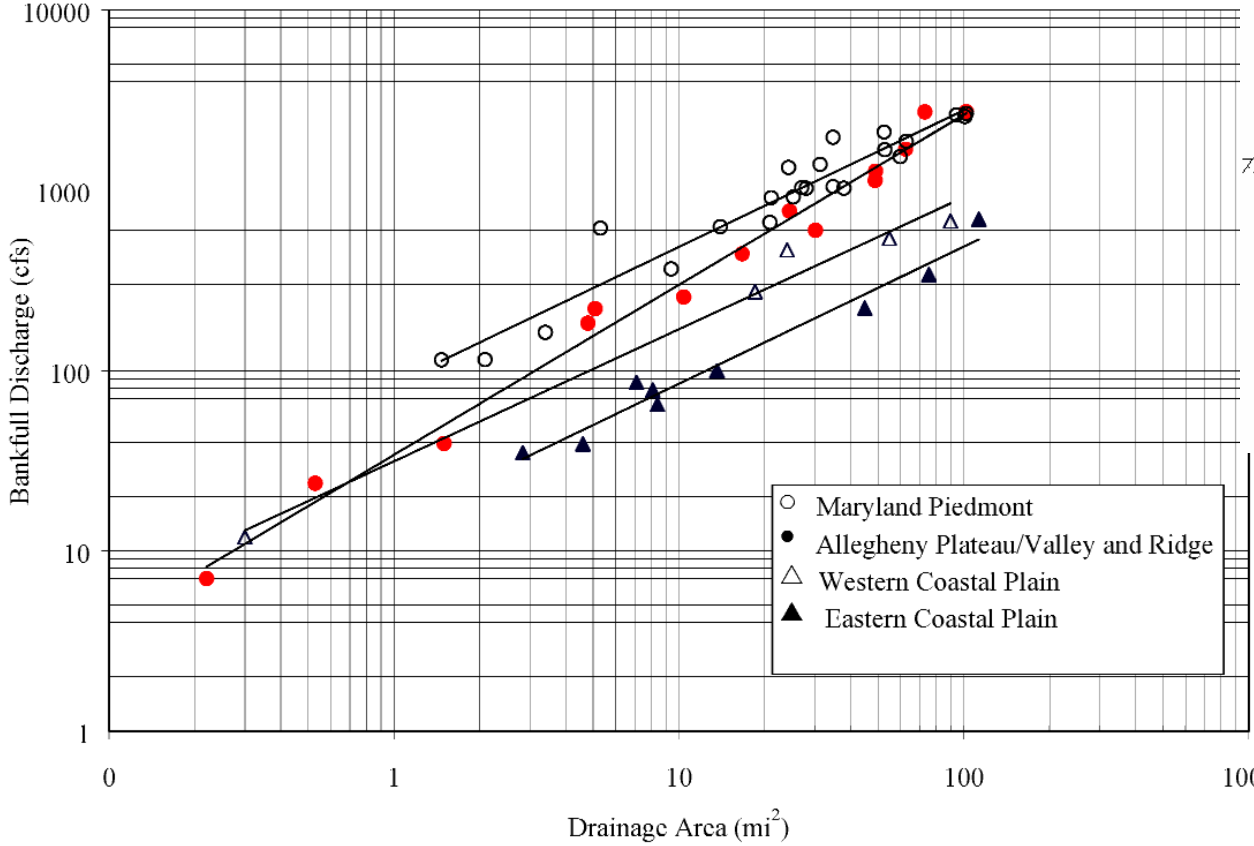
Later studies show a much narrower range of return interval for bankfull flow …

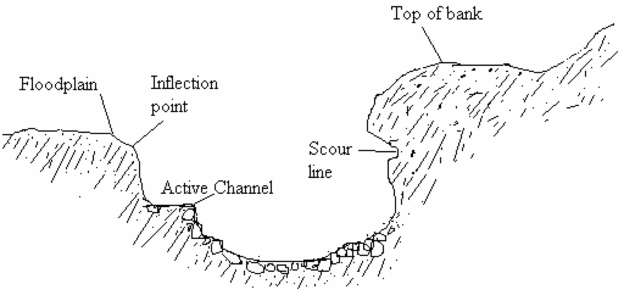
Figure 11. Frequency of recurrence interval for field-estimated bankfull discharge.

MARYLAND STREAM SURVEY: BANKFULL DISCHARGE AND CHANNEL CHARACTERISTICS OF STREAMS IN THE COASTAL PLAIN HYDROLOGIC REGION

U.S. Fish & Wildlife Service
Chesapeake Bay Field Office

CBFO-S03-02





MARYLAND STREAM SURVEY: BANKFULL DISCHARGE AND CHANNEL CHARACTERISTICS OF STREAMS

U.S. Fish & Wildlife Service, Chesapeake Bay Field Office

APPENDIX B – PROTOCOLS FOR FIELD SURVEYS AT GAGE STATIONS - DETERMINATION OF BANKFULL STAGE

Steep, confined streams in rocky canyons often lack distinguishable floodplains, so other features must be used. Where floodplains are absent or poorly defined, other indicators may serve as surrogates to identify bankfull stage.

Useful indicators include: Top of Point Bars, Change in Vegetation, Change in Slope, Change in Bank Materials, Bank Undercuts, Stain Lines.

Deposits of pine needles, twigs, and other floating materials are common along streams, but they are seldom indicators of bankfull stage.

If stream gage data is available for the stream, observations of indicators at or near the gages may help to identify the indicators most useful for a particular area.

Bankfull discharges tend to have similar flow-frequency (approximately 1.5 years) … among sites in a given climatic region. Use … observations of bankfull stage at local stream gages to test the reliability of the various indicators for your geographic area. Compare your calculation of bankfull discharge to the regional averages. If it is different, refer to the USGS peak flow procedures for the area to determine if a significantly different area-runoff relationship exists. In the absence of other reasonable explanations, examine your methods.

The *hypothesis* that the bankfull channel is adjusted to a flood of return interval 1.0 - to 1.5 yr is now used to *select the indicators* for the "bankfull" channel. ***The logic has come full circle!*** The appropriate bankfull indicator is taken to be that corresponding to a flow with a return interval in the range 1.0 – 1.5 yr. Regardless of its relevance to channel morphology! No wonder data on the flood frequency of bankfull channels have become quite tidy… it no longer makes a difference whether one is looking at the top of the bank!

→We are no longer looking for the bankfull channel, but for the channel that holds the 1.0 – 1.5 yr flood.

→One could reasonably ask … for what purpose? If the bankfull channel is to be used to size a design channel, and the new "bankfull" is nothing more than the 1.0 – 1.5 yr flood, ***then why not use that return interval as the design criteria***?

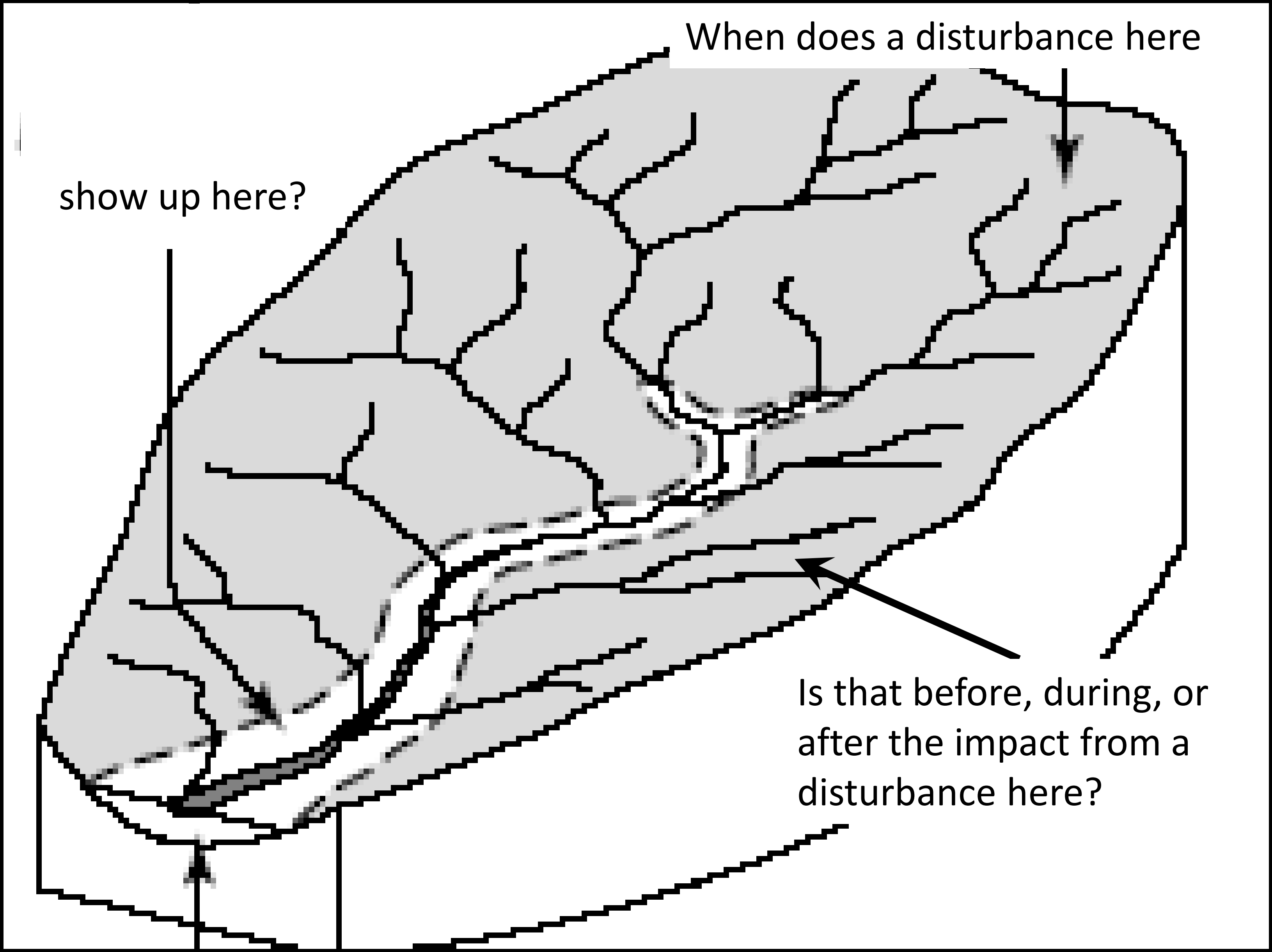
→ One could reasonably say, "I want my channel to flood two years out of three" (the 1.5 yr flood). {or every other year: the 2-yr flood. or four years out of five: the 1.25 yr flood}. There is good reason for considering flood frequency in channel design – but why jump on the bankfull track, which only takes you in circles? Just specify the desired flood frequency and go with it!

Why is there a focus on equilibrium channel geometry and its bankfull flow?

To provide a template
a channel geometry for design

There are a couple of problems …

Where is equilibrium found in a real watershed?



When does a disturbance here

show up here?

Is that before, during, or after the impact from a disturbance here?

In many cases, there is no steady state, & there is no template

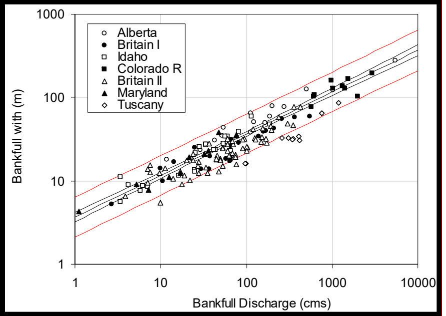
But there is a more fundamental problem!

At the core of the template approach is a ***correlation*** between channel geometry, flow, and sediment supply

This correlation ***is*** remarkable:

The flow that moves the most sediment, over time, tends to just fill the channel and occurs ever year or few.

The width of channels increases very consistently with the square root of discharge.



The correlation ***requires*** that the channels have ***adjusted*** to their water and sediment supply.

But what if channel is currently adjusting, or perpetually adjusting?

How would you know?

!

A template approach provides no basis for ***linking cause and effect*** in a logically complete and ***testable*** framework.

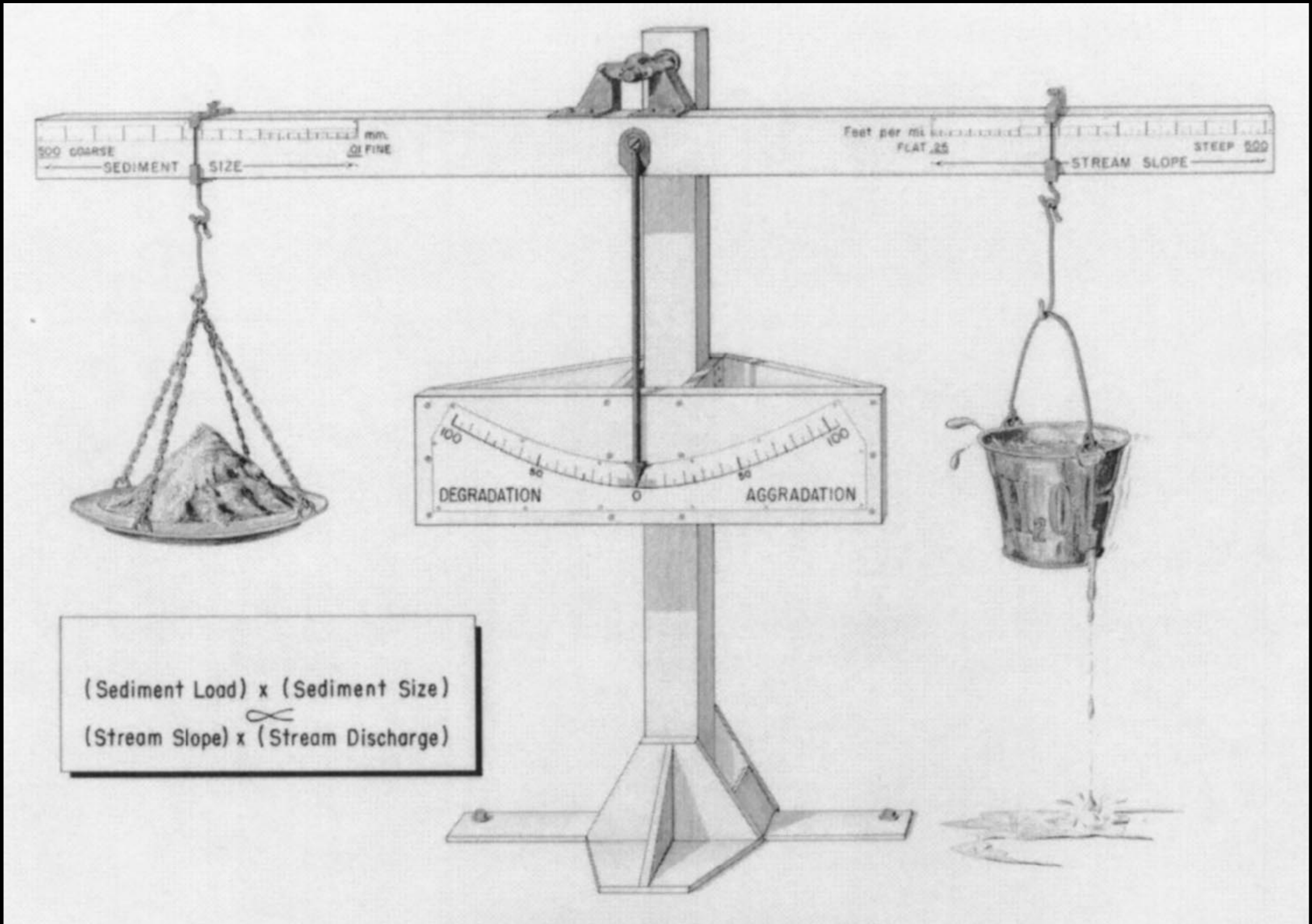
If a template-designed project "fails",
how is the method to be improved?

Understanding Channel Change

linking sediment supply to transport ***capacity***

with flow ***competence*** *invoked when needed*

Lane/Borland "Stable Channel" Balance (USBR 1955-1960)



STABLE CHANNEL BALANCE

Sediment Supply Transport Capacity

This defines the controls for ***an alluvial channel***

The Lane Balance, quantified almost 40 yrs ago by Henderson (1966, Open Channel Flow)

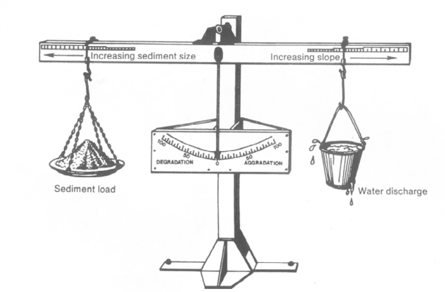
$$\begin{tikzcd}
\text{Einstein-Brown} & & \text{depth-slope} & & \text{continuity} & & \text{Chezy} \\
q^* \propto (\tau^*)^3 \arrow[dr] & & \tau \propto RS \arrow[ddl] & & q \propto UR \arrow[dr] & & U \propto \sqrt{RS} \arrow[dl] \\
& q_b \propto \dfrac{\tau^3}{D^{3/2}} \arrow[d] & & & & q \propto R^{3/2}\sqrt{S} \arrow[d] & \\
& q_b \propto \dfrac{(RS)^3}{D^{3/2}} \arrow[drr] & & & & R^3 \propto \dfrac{q^2}{S} \arrow[dll] & \\
& & & q_b \propto \dfrac{q^2 S^2}{D^{3/2}} & & &
\end{tikzcd}$$

or

$$\boxed{S \propto \frac{\sqrt{q_b}\, D^{3/4}}{q}}$$

or for two cases

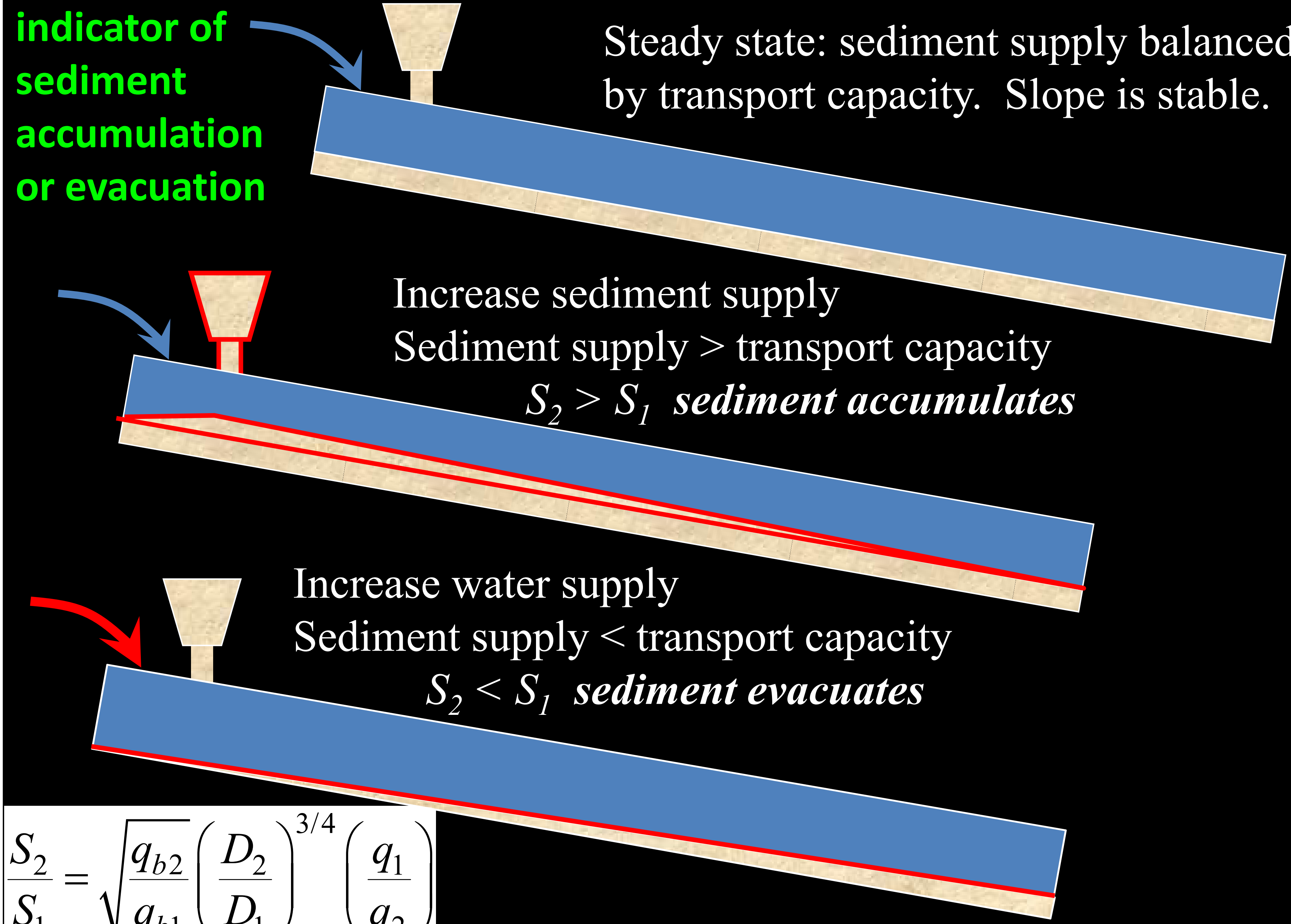
$$\frac{S_2}{S_1} = \sqrt{\frac{q_{b2}}{q_{b1}}}\left(\frac{D_2}{D_1}\right)^{3/4}\left(\frac{q_1}{q_2}\right)$$



What if q_b increases and D decreases? Lane's balance is indeterminate.

Interpretation, for evaluating stream behavior

Slope is indicator of sediment accumulation or evacuation



Steady state: sediment supply balanced by transport capacity. Slope is stable.

Increase sediment supply

Sediment supply > transport capacity

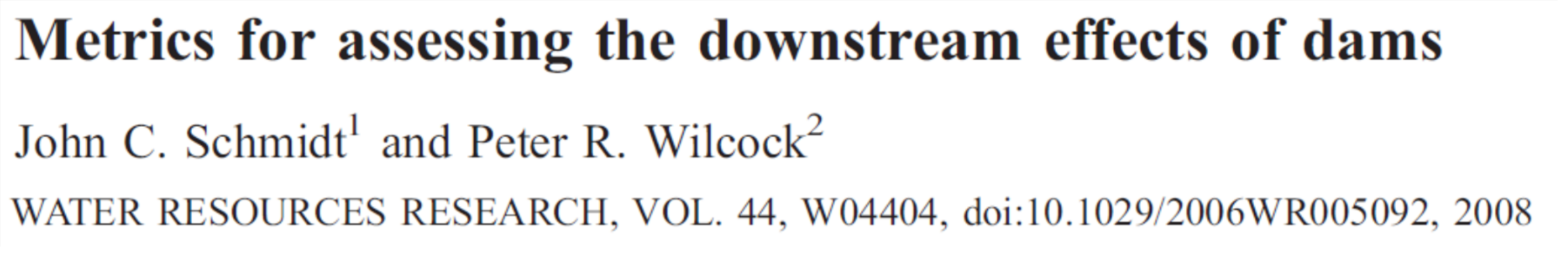
$S_2 > S_1$ ***sediment accumulates***

Increase water supply

Sediment supply < transport capacity

$S_2 < S_1$ ***sediment evacuates***

$$\frac{S_2}{S_1} = \sqrt{\frac{q_{b2}}{q_{b1}}}\left(\frac{D_2}{D_1}\right)^{3/4}\left(\frac{q_1}{q_2}\right)$$



Metrics for assessing the downstream effects of dams

John C. Schmidt[1] and Peter R. Wilcock[2]

WATER RESOURCES RESEARCH, VOL. 44, W04404, doi:10.1029/2006WR005092, 2008

$$\frac{S_{post}}{S_{prw}} = \sqrt{\frac{Q_{spost}}{Q_{spre}}} \left(\frac{D_{post}}{D_{pre}}\right)^{3/4} \left(\frac{Q_{pre}}{Q_{post}}\right)$$

← Surplus or Deficit?

Capacity *v.* Supply

$$\tau^* \propto \frac{h_{post} S_{pre}}{D_B}$$

← If Deficit, will channel scour?

Flow Competence !!!

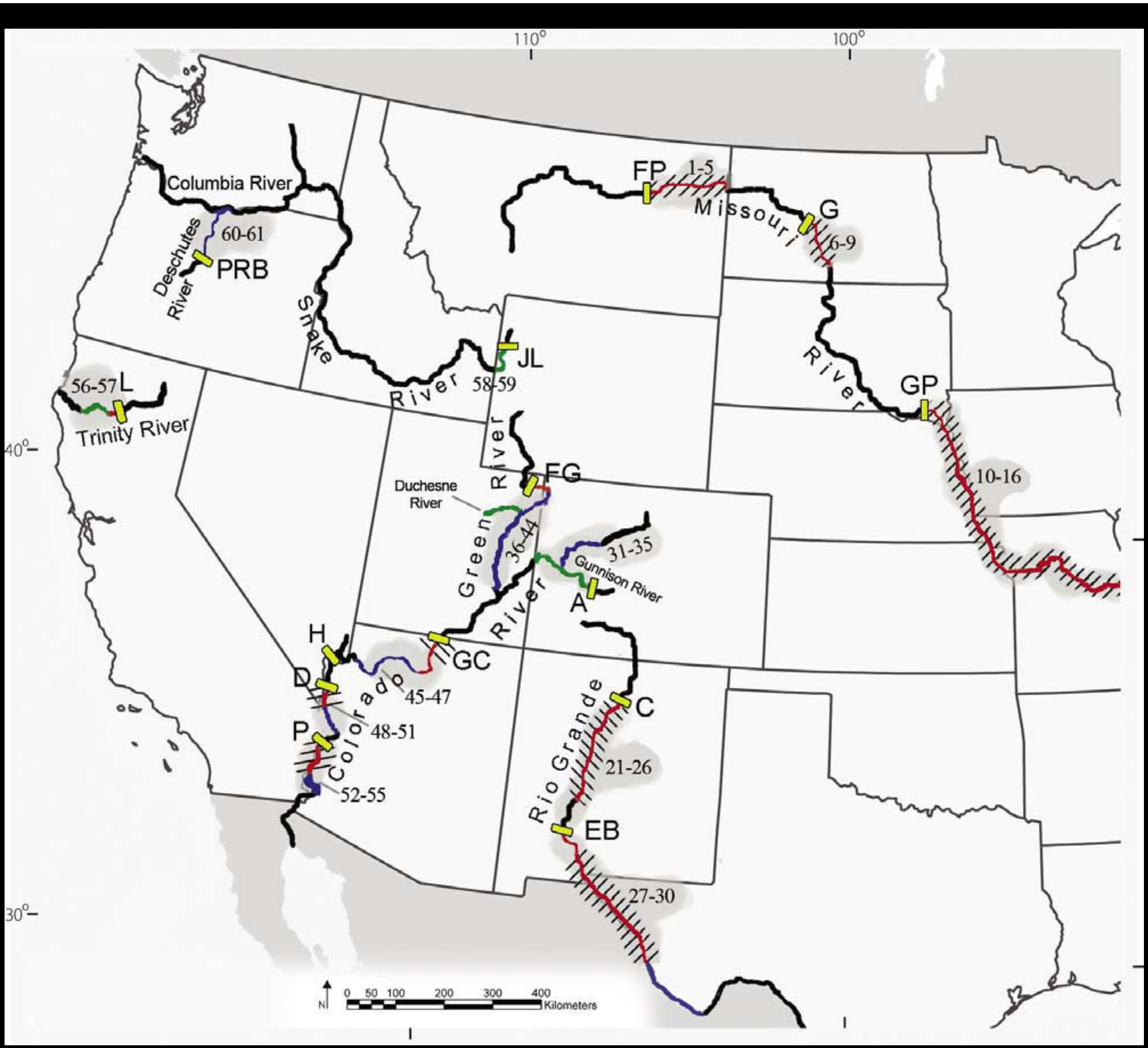


Figure 4. Location of rivers, dams, and reaches summarized in this paper. Numbers refer to reaches listed in Table 1. Red indicates reaches in sediment deficit, green indicates reaches in surplus, and blue indicates reaches where S* is indeterminate, as listed in Table 1. Cross-hatched reaches are those where t* > 0.06 and incision is likely. Dams are FP, Fort Peck; G, Garrison; GP, Gavins Point; C, Cochiti; EB, Elephant Butte; A, Aspinal Unit of three dams; FG, Flaming Gorge; GC, Glen Canyon; H, Hoover; D, Davis; P, Parker; L, Lewiston; JL, Jackson Lake; and PRB, Pelton–Round Butte complex of dams.

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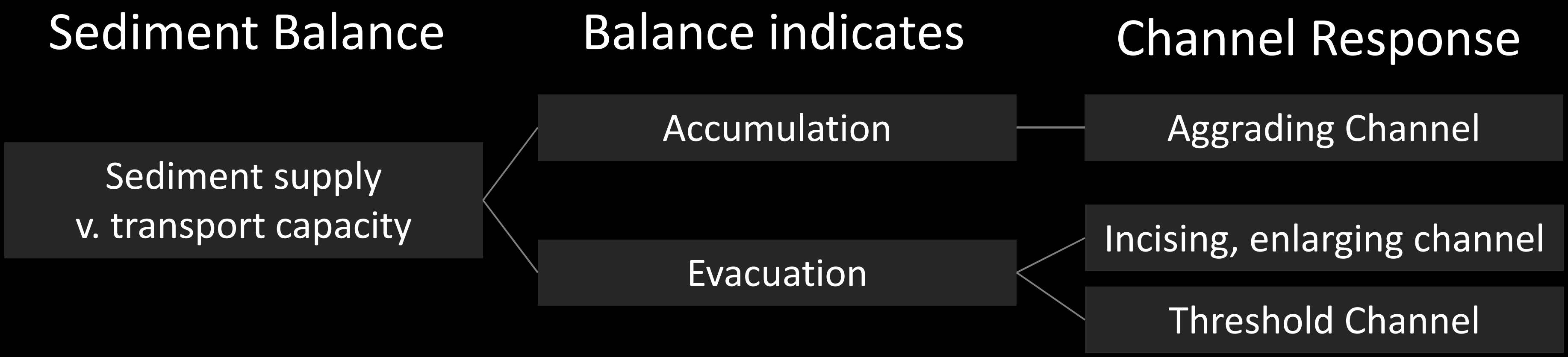
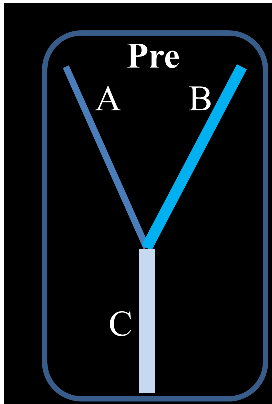
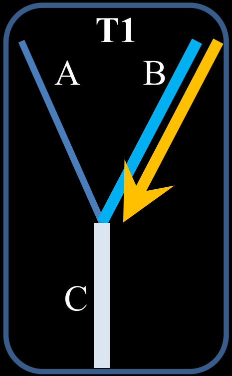
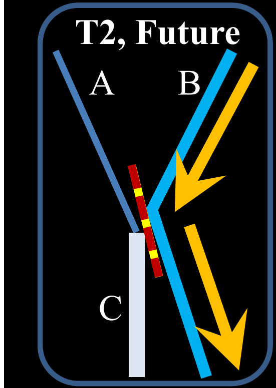


Sediment Balance	Balance indicates	Channel Response
Sediment supply v. transport capacity	Accumulation	Aggrading Channel
	Evacuation	Incising, enlarging channel Threshold Channel

(1)Sediment Balance
(2)Threshold Evaluation
(3)How fast?







CHALLENGE PROBLEM: What is the future condition of Reach C?

Reach A and Reach B combine to form Reach C. Gravel bed Rivers.

Discharge in Reach A is about 30% discharge in Reach B.

The rate of sediment supply from A and B is unknown but we can suppose that sediment supply of A is smaller than B, perhaps as small as 25% of that from B. Field inspection indicates that the grain size of sediment supply is comparable from both tributaries.

Time 1: Reach B experiences an enormous increase in sediment supply, perhaps 10 to 50 times the natural supply. The supply contains much more silt and sand than the natural supply. This supply will last for many years.

Time 2: To protect Reach C from the impacts of the increased sediment supply, Reach B is diverted from Reach C. Now Reach C carries only the inputs from Reach A. What is short term response?

Future: same conditions as Time 2, but a decade or two into the future.

Period	History
Pre	$Q_A \approx 0.3\, Q_B$, $Q_{sA} = \beta Q_{sB}$ $(0.25 < \beta < 1)$, $D_A \approx D_B$
T1	Very large sediment loading to River B
T2, Future	B diverted from A + B

$$\frac{S_2}{S_1} = \sqrt{\frac{q_{b2}}{q_{b1}}}\left(\frac{D_2}{D_1}\right)^{3/4}\left(\frac{q_1}{q_2}\right)$$

Calculate S_2/S_1 *for Reach C*

Period	Discharge Q	Sediment Supply Q_s	Sediment Supply D	Channel Width b
Pre	$Q_A + Q_B$	$Q_{sA} + Q_{sB}$	?*	100 m
T1	$Q_A + Q_B$	$Q_{sA} + \alpha Q_{sB}$	Finer	150 m
T2	Q_A	Q_{sA}	Pre	75 m
Future	Q_A	Q_{sA}	Pre	?**

Handy Math

$0.3 / 1.3 = 0.23$

$\sqrt{0.23} = 0.48$

* We do not need to know the grain size of the pre-disturbance sediment supply, only the ***change*** in its grain size

** Use the downstream hydraulic geometry, which says that the width varies with the square root of discharge

A transport function.

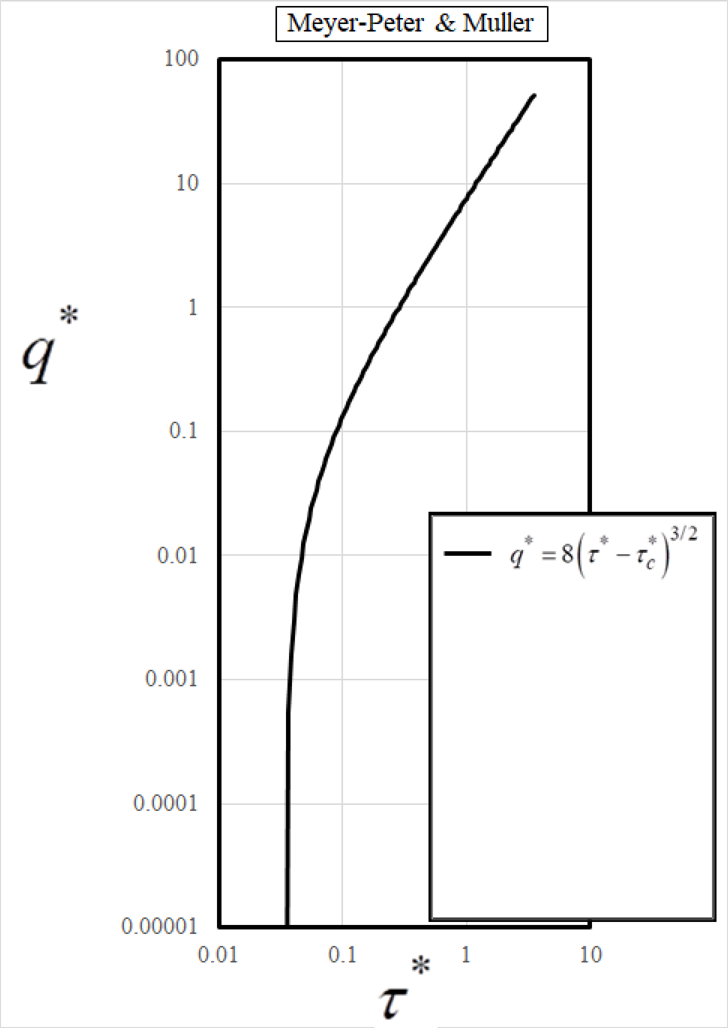
As long as boundary stress τ_o is defined as the grain stress, they are all pretty much the same.

In addition to finding the grain stress, the big challenges are

(i) Determining the appropriate critical stress τ_c

(ii) Accounting for natural complexity in channel geometry, flow, and bed material

(iii) Determining whether much of the load is not bed material in your section



$$q^* = \frac{q_b}{\sqrt{(s-1)gD^3}}$$

$$\tau^* = \frac{\tau_o}{(s-1)\rho g D}$$

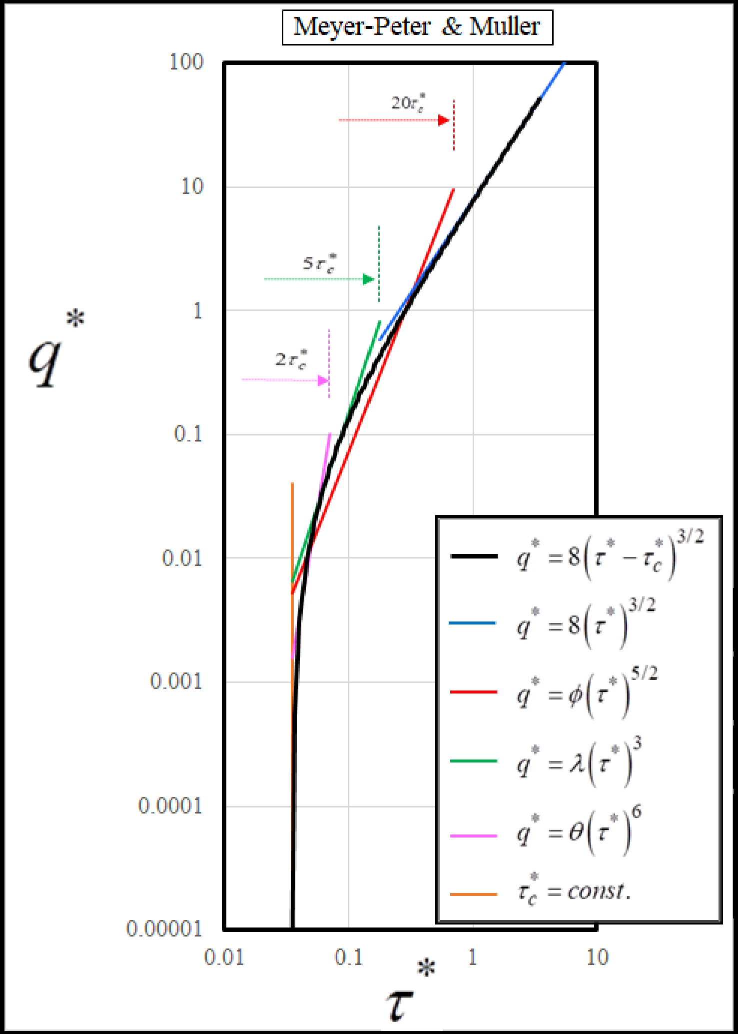
$$\tau_c^* = \frac{\tau_c}{(s-1)\rho g D}$$

q_b	unit transport rate L^2 / T
τ_o	boundary stress M / [LT^2]
g	gravity M / L^2
ρ	water density M / L^3
s	ρ_s / ρ rel. sediment density
D	grain size L

Different formulas used in practice provide a loglinear approximation to parts of the transport function.

(It is convenient to eliminate the curvy part due to τ_c^*)

Let's put these loglinear approximations in the format of our four sediment balance variables: q_b, D, S, q. To go to geomorphic cause and effect, think of sediment supply q_b, D and water discharge q as provided by the watershed , and slope S is the dependent variable.



Limit at $\tau^* >> \tau_c^*$ (finest sands)

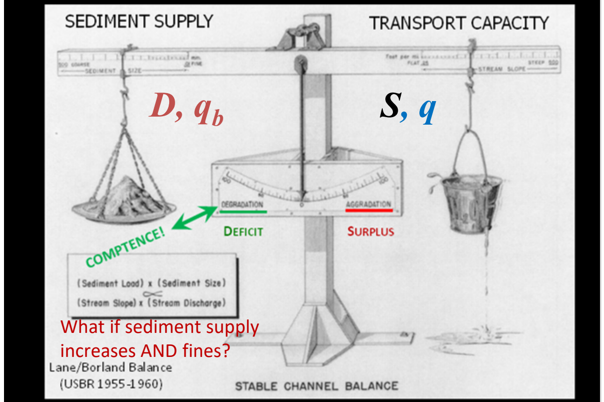
Engelund-Hansen – total load of sand

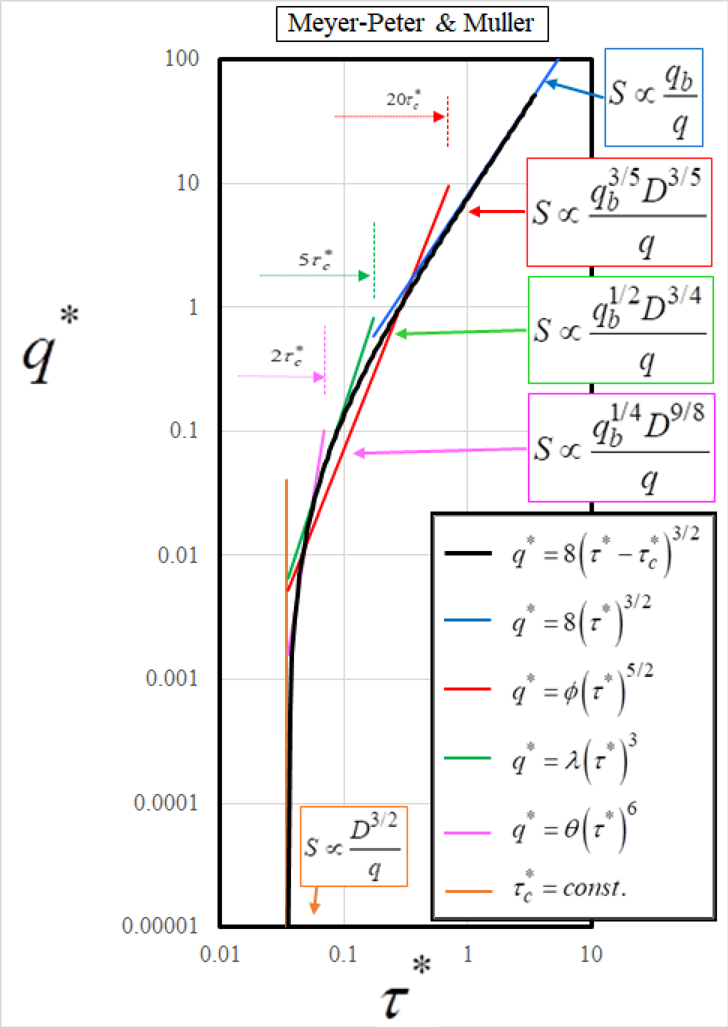
Einstein-Brown – all sizes?

Approx. for $\tau^* < 2\tau_c^*$

Threshold conditions (coarse gravels)

Let's put these loglinear approximations in the format of the four sediment balance variables.





$$q^* = \frac{q_b}{\sqrt{(s-1)gD^3}}$$

$$\tau^* = \frac{\tau_o}{(s-1)\rho g D}$$

Using depth-slope product

$\tau_o \propto hS$

and Chezy flow resistance

$q = Uh \propto h^{3/2}\sqrt{S}$

we can eliminate τ_o and h, leaving our 4 sediment balance variables q, q_b, D, S

As we go from **finest sand** to **coarsest gravel**, the dependence of slope S on D and q_b entirely changes!!!

$$q^* \propto \tau_*^b$$

$$q_s \propto h^b S^b D^{(3/2-b)}$$

$$q = Uh \propto h^{3/2}\sqrt{S}$$

$$h^b \propto q^{\frac{2b}{3}} S^{\frac{-b}{3}}$$

$$q_s \propto q^{\frac{2b}{3}} S^{\frac{-b}{3}} S^b D^{(3/2-b)}$$

$$q_s \propto q^{\frac{2b}{3}} S^{\frac{2b}{3}} D^{(3/2-b)}$$

$$S \propto q_s^{\frac{3}{2b}} q^{-1} D^{-\left(\frac{9}{4b}-\frac{3}{2}\right)}$$

$$\tau_c^* = \text{const} \propto \frac{hS}{D}$$

$$h \propto \left(\frac{q^2}{S}\right)^{1/3}$$

$$\text{const} \propto \frac{(qS)^{2/3}}{D}$$

$$S \propto \frac{D^{3/2}}{q}$$

Coarsest gravel:
S depends on grain size D and does not depend on sediment supply rate q_b

Finest sand:
S depends on concentration and does not depend on grain size D

Slope change is slow, but provides a useful steady state **target**, or reference, toward which channel will evolve in response to changes in water and sediment supply. It may well never get there (slope change can take centuries, even millennia) before water and sediment supply change yet again.

In the immediate future:

>> a greater slope needed to transport the supplied sediment with the available flow indicates conditions of sediment accumulation, a surplus

>> a smaller slope needed to transport the supplied sediment with the available flow indicates conditions of sediment evacuation*, a sediment deficit.

* A smaller slope indicates sediment deficit, but does not necessarily indicate degradation (incision). If the channel bed material is coarser than the sediment supply, the flow may not be able to mobilize the bed, even if it is able to transport all the supplied sediment. This illustrates the difference between ***flow competence & transport capacity!***

Detection of sediment surplus or deficit does ***not*** tell the whole story of channel change. Rather, it provides a key detail in setting up the story.

We started with:

how will a channel change in response to changes in water and sediment supply.

Now we ask:

how will a channel change in response to changes in water and sediment supply
that produce a sediment surplus (or deficit)?

I think this is a big deal!

Surplus: more frequent flooding, larger bars → more active channel shifting, increased potential for avulsion

Deficit: potential for incision, less frequent flooding, smaller bars → less active channel shifting